Time Series Econometrics

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ECON30401 Video Exercise 2

Semester 1 2016

1. The $AR(2)$ process is

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

where ε_t is a mean zero i.i.d process with variance σ^2.

(a) Derive formulae for the predictors of Y_{T+1}, Y_{T+2} and given outcomes for the process to time T. Your expressions should involve only the parameters μ, ϕ_1, ϕ_2 and the sample outcomes $y_T, y_{T-1}, y_{T-2}, ...y_1)$. *[Hint: The best predictor is the conditional mean* $\mathbb{E}[Y_{T+i} | \mathcal{I}_T]]$ *for* $i = 1, 2, 3$.*]*

(b) The forecast error (FE(i)) for an i-step ahead prediction is defined the conditional variance

$$FE(i) = \mathbb{E}\left[(Y_{T+i} - \mathbb{E}[Y_{T+i} \middle| \mathcal{I}_T])^2 \, |\mathcal{I}_T\right]$$

Show that the $AR(2)$ has

$$\begin{aligned} FE(1) &= \sigma^2 \\ FE(2) &= \sigma^2(1 + \phi_1^2). \end{aligned}$$

(c) Without doing any further formal analysis, what do you think will happen to $FE(i)$ as i increases for this $AR(2)$? What does this imply for the expected accuracy of forecasts as the horizon i increases?

In question 2 D_{qt} is a seasonal dummy taking value 1 when t is quarter q and 0 otherwise. Finally ε_t is white noise with variance σ^2.

2(a) Consider the following Seasonal ARMA(2,1) model for quarterly data

$$Y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \phi_2 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \tag{1}$$

where $|\phi_2| < 1$.Derive the mean of Y_t in Quarters 1,2,3,4 (denoted $\mu_1, \mu_2, \mu_3, \mu_4$ respectively).

Hint: Use the fact the quarterly means are equal over time under the WN assumption and $|\phi_2| < 1$.

2(b) Derive the variance of Y_t^{SA}

$$Y_t^{SA} = Y_t - (\alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \phi_2 Y_{t-2}).$$