ECON30401 Time Series Econometrics

Lecture 4: Prediction & Seasonality

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Recap so Far

- Theoretical Properties of ARMA Processes
- Estimation & Inference with Sample Data
- Fundamental & Broad Time Series Concepts
- Move on to some more specialist (yet still rather broad) topics..

Part I: Prediction

How to predict outcome of process in future periods?

- Sample data of size T up to today
- Estimate true process
- Construct 'best' guess (prediction) of tomorrow

Example: BoE Forecast of CPI

Example: High-freq. traders predict stock price tomorrow.

Part II: Seasonality

Economic Time Series Process often display repeated patterns

C:/Users/msassng4/Desktop/Teachi

Implications for inference?

How to model seasonality?

Prediction

Observe sample up to $y_1, .., y_T$ up time T (today)

Came from some process Y_t

How to predict outcomes at $T + 1, T + 2, ..$ (future)

How to measure precision of the prediction (i.e our guess)

Prediction from AR(1)

$$Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim WN(\sigma^2)$$

Obvious guess at T+1 given $y_1, .., y_T$? $\hat{Y}_{T+1} = \mu + \phi_1 y_T$

Why? If $\mathbb{E}[\varepsilon_{T+1} | Y_T = y_T] = 0$

[Slightly Stronger Assumption than ε_t Serially Uncorrelated]

$\Rightarrow$ $\mathbb{E}[Y_{T+1} | Y_T = y_T] = \mu + \phi_1 y_T$

- In practise use OLS estimates of μ, ϕ_1 to form $\hat{Y}_{T+1}$

Imprecision of Prediction from AR(1)

$$\hat{Y}_{T+1} = \mu + \phi_1 y_T$$

$$Y_T = y_T \Rightarrow Y_{T+1} = \mu + \phi_1 y_T + \varepsilon_{T+1}$$

Forecast Error: $Y_{T+1} - \hat{Y}_{T+1} = \varepsilon_T$

Forecast Error Variance $Var(\varepsilon_{T+1}) = \sigma^2$

Second Step Ahead Prediction from AR(1)

$$Y_{T+2} = \mu + \phi_1 Y_{T+1} + \varepsilon_{T+2}$$

- Do not observe y_{T+1} so use best guess $\hat{Y}_{T+1}$

$$\hat{Y}_{T+2} = \mu + \phi_1 \hat{Y}_{T+1}$$

Why? If $\mathbb{E}[\varepsilon_{T+2} | Y_T = y_T] = 0$

Forecast

$$\begin{aligned} \mathbb{E}[Y_{T+2} | Y_T = y_T] &= \mu + \mathbb{E}[Y_{T+1} | Y_T = y_T] \\ &= \mu + \phi_1 \hat{Y}_{T+1} \\ &= = \mu + \phi_1(\mu + \phi_1 y_T) \end{aligned}$$

Forecast Error: $Y_{T+2} - \hat{Y}_{T+2} = \varepsilon_{T+2} + \phi_1 \varepsilon_{T+1}$

J Step Ahead Prediction from AR(1)

Predict Y_{T+J} for $J \geq 1$

Best guess? If $\mathbb{E}[\varepsilon_{T+J}|Y_T = y_T] = 0$

Forecast

$$\begin{aligned} \hat{Y}_{T+J} &= \mathbb{E}[Y_{T+J}|Y_T = y_T] \\ &= \mu + \phi_1 \hat{Y}_{T+J-1} \end{aligned}$$

Forecast Error: $Y_{T+J} - \hat{Y}_{T+J} = \sum_{j=1}^{J} \phi_1^{J-j} \varepsilon_{T+j}$

Forecast Error Variance $\sigma^2 \sum_{j=1}^{J} \phi_1^{2(J-j)}$

$J \to \infty$ Step Ahead Prediction from AR(1)

What happens as $J \to \infty$ if $|\phi_1| < 1$

Forecast $\hat{Y}_{T+J} \to \frac{\mu}{1-\phi_1}$

Forecast Error Variance $\sigma^2 \sum_{j=1}^{J} \phi_1^{2(J-j)} \to \frac{\sigma^2}{1-\phi_1^2}$

General AR(p) Predictors

Prediction from general AR(p) models follows similarly

Video Exercise 2 considers $p = 2$

In practise we use OLS estimates of coefficients

$\Rightarrow$ further forecast error

- Only need to derive properties of predictors with coefficients known.
- Will not consider prediction of MA or ARMA

Seasonality and Seasonal Adjustment

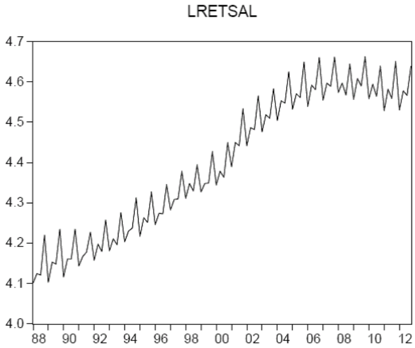
- Seasonality is difficult to define in a simple way
  but its manifestation is patterns that repeat over years
- In macroeconomics:
  - Retail sales increase prior to Christmas
  - Industrial production drops in the summer
  - Some food prices increase in winter
- Statistical agencies produce *seasonally adjusted series*
- Seasonal adjustment makes implicit assumptions about seasonality

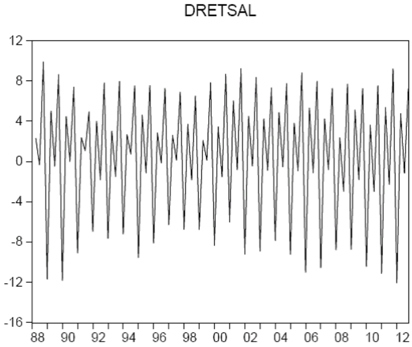
  Adjustment can have undesirable side-effects
  May prefer to use seasonally unadjusted data
- Seasonality often considered as **deterministic** and/or **stochastic**

Example: Retail Sales

- Volume of retail sales, predominantly food stores, 1988Q1-2012Q4

Log series & growth rate (%)





Deterministic Seasonality

Deterministic seasonality seasonal dummy variables

Quarterly example: 3 seasonal dummy variables

$$Y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \varepsilon_t$$

D_{qt} takes value 1 for quarter q & zero otherwise
(arbitrary which seasonal dummy to exclude)

- Implication:

$$\mathbb{E}[Y_t \,|q = 1] : \mu_1 = \alpha_0 + \alpha_1$$
$$\mathbb{E}[Y_t \,|q = 2] : \mu_2 = \alpha_0 + \alpha_2$$
$$\mathbb{E}[Y_t \,|q = 3] : \mu_3 = \alpha_0 + \alpha_3$$
$$\mathbb{E}[Y_t \,|q = 4] : \mu_4 = \alpha_0$$

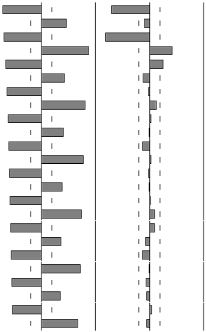
NB: Beware the "dummy variable trap"!

Correlogram with Deterministic Seasonality I

Correlogram of DRETSAL

Date: 02/14/13 Time: 09:04
Sample: 1988Q1 2012Q4
Included observations: 99

Autocorrelation | Partial Correlation



	AC	PAC	Q-Stat	Prob
1	-0.727	-0.727	53.878	0.000
2	0.483	-0.095	77.943	0.000
3	-0.704	-0.830	129.51	0.000
4	0.917	0.439	218.09	0.000
5	-0.674	0.259	266.42	0.000
6	0.449	-0.116	288.13	0.000
7	-0.652	-0.012	334.27	0.000
8	0.850	0.135	413.72	0.000
9	-0.630	0.031	457.75	0.000
10	0.421	-0.007	477.63	0.000
11	-0.622	-0.126	521.56	0.000
12	0.808	0.030	596.65	0.000
13	-0.601	-0.015	638.60	0.000
14	0.409	-0.005	658.24	0.000
15	-0.596	0.017	700.56	0.000
16	0.777	0.092	773.24	0.000
17	-0.574	0.091	813.46	0.000
18	0.380	-0.073	831.33	0.000
19	-0.574	-0.128	872.45	0.000
20	0.748	-0.007	943.29	0.000
21	-0.551	-0.064	982.19	0.000
22	0.365	-0.053	999.46	0.000
23	-0.545	0.047	1038.5	0.000
24	0.707	-0.047	1105.2	0.000

Deterministic Seasonal Adjustement

$$Y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \varepsilon_t$$

Find strong dependence in Y_t

Seasonal dependence is not focus of interest

Seasonally Adjust data by removing $\alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t}$

Estimate via OLS

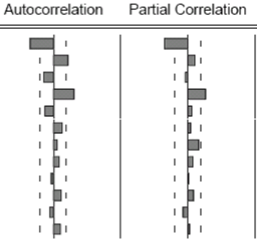
Correlogram of Seasonally Adjusted Data

- Remove quarterly means through prior regression:

  Residuals from $Y_t = \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \alpha_4 D_{4t} + \varepsilon_t$

Correlogram of DEMEAN

Date: 02/04/13 Time: 17:55
Sample: 1988Q1 2012Q4
Included observations: 99



	AC	PAC	Q-Stat	Prob
1	-0.357	-0.357	13.027	0.000
2	0.225	0.111	18.234	0.000
3	-0.139	-0.033	20.260	0.000
4	0.312	0.272	30.517	0.000
5	-0.139	0.065	32.570	0.000
6	0.135	0.041	34.530	0.000
7	0.060	0.171	34.921	0.000
8	0.088	0.082	35.780	0.000
9	-0.031	0.014	35.889	0.000
10	0.120	0.085	37.499	0.000
11	-0.054	-0.068	37.829	0.000
12	0.111	0.031	39.233	0.000

Stochastic Seasonality

Seasonality may be random e.g quarterly seasonaliy that dies away

Stochastic seasonality operates through *ARMA* dynamics

- Quarterly seasonal $AR(1)$:

$$Y_t = \alpha + \phi_4 Y_{t-4} + \varepsilon_t$$

Stationary if $|\phi_4| < 1$

Stochastic Seasonality: MA Representation

- Stationary quarterly *SAR*(1)

$$\begin{aligned} Y_t &= \alpha + \phi_4 Y_{t-4} + \varepsilon_t \\ &= \frac{\alpha}{1-\phi_4} + \varepsilon_t + \phi_4 \varepsilon_{t-4} + \phi_4^2 \varepsilon_{t-8} + \dots \\ &= \mu + \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i} \end{aligned}$$

- At *seasonal lags*

$$\begin{aligned} \theta_4 &= \phi_4, \ \theta_8 = \phi_4^2, \ \text{etc} \\ \theta_i &= 0, \ i = 1, 2, 3, 5, 6, 7, 9, \dots \ \text{etc} \end{aligned}$$

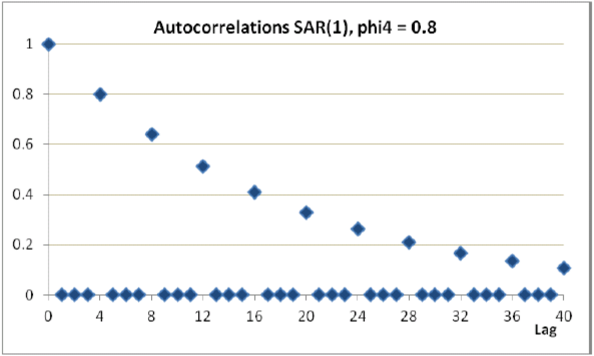
- Taking expectations

$$\mathbb{E}[Y_t] = \frac{\alpha}{1-\phi_4} \quad \text{all } t$$

Constancy over quarters may be implausible

Stochastic Seasonality: Autocorrelations II

- With $\phi_4 = 0.8$, autocorrelation function for $SAR(1)$ process:



More General Seasonal Models

Can generalise $SAR(1)$ in various ways

- Sometimes also SMA, eg quarterly $SARMA(1,1)$

$$Y_t = \alpha + \phi_4 Y_{t-4} + \varepsilon_t + \theta_4 \varepsilon_{t-4}$$

- Can combine deterministic & stochastic e.g seasonality

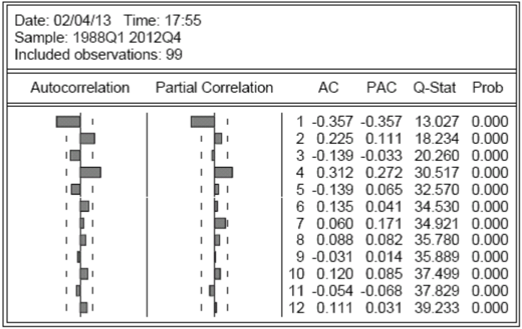
$$Y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \phi_1 Y_{t-1} + \phi_4 Y_{t-4} + \varepsilon_t$$

In practice, often include seasonal dummies & consider other patterns

Example: Retail Sales Again

- After removing seasonal means (shown again)

Correlogram of DEMEAN



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Included observations: 99

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
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Model for Retail Sales Growth

Dependent Variable: DRETSAL
Method: Least Squares
Sample (adjusted): 1989Q2 2012Q4
Included observations: 95 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.051583	0.007618	6.771151	0.0000
@SEAS(1)	-0.091805	0.016755	-5.479391	0.0000
@SEAS(2)	-0.049281	0.009397	-5.244414	0.0000
@SEAS(3)	-0.046960	0.009335	-5.030586	0.0000
DRETSAL(-1)	-0.292614	0.093266	-3.137396	0.0023
DRETSAL(-4)	0.287895	0.094389	3.050080	0.0030

Intercept largest in fourth quarter

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.447242	Prob. F(4,85)	0.7741
Obs*R-squared	1.958219	Prob. Chi-Square(4)	0.7434

What have we Learnt today?

Methods to predict from AR(1)

Measures of Forecast Uncertainty

Discussed seasonality

Issues seasonality brings for time series analysis

Methods of modelling Seasonality & Seasonal Adjustment

End of My Half of the Course

End of the first half of lectures

...PC Lab next week in Eviews related to Lect 1-4

Move on to Multivariate models and Unit Roots with Prof. Hall

Repeat: 4 more lectures, 2 exercise classes and a PC Lab

Crucial: Recap basic Linear Algebra- Rank, Inverse, Matrix Addition/Multiplication.