ECON30401 Time Series Econometrics

Lecture 3 Estimation, Hypothesis Testing & Model Selection

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16th October 2015

Recap so Far

- Distinction between **Process** Y_t and **Sample Realisation** y_t
- White Noise & **Stationarity**
- Measuring Dependence- **Auto-Correlation Function(ACF)**
- Properties of **ARMA** Models

Studied Theoretical Properties of ARMA Processes

Motivation For Today's Lecture

- 'Unknown Process Y_t generated data we observe $\{y_t\}_{t=1}^{T}$
- Data should reflect properties of this Process
- How to infer properties of Y_t from the data?

Methods to estimate, test and select between models

What We Will Cover

1. Sample Autocorrelation Function & Testing
2. Interpeting Eviews Correlogram
3. Estimating ARMA Models & Testing
4. Testing Serial Correlation (Video)
5. Model Selection- Information Criterion

Overview of Problem

Y_t unknown. **Example**: $Y_t = 1.1 + 0.5Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \text{WN}(1)$

Theoretical Properties: $\rho(k) = Corr(Y_t, Y_{t-k}) = 0.5^k$

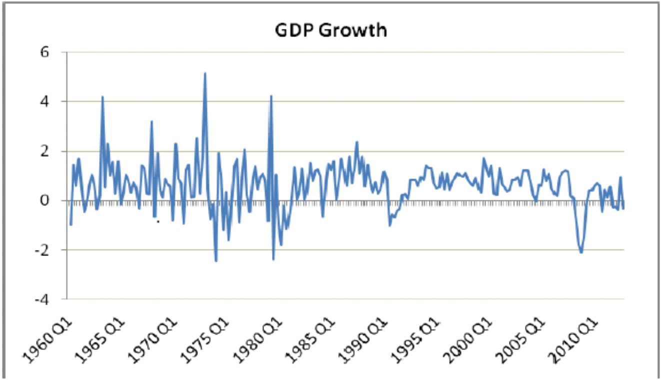
Generates the data we observe $y_1, .., y_T$

Data should reflect these properties...

How to estimate correlations and test hypotheses?

How to determine process is $Y_t = 1.1 + 0.5Y_{t-1} + \varepsilon_t$?

Empirical Problem: UK GDP Growth



Which Process Generated UK GDP Growth?...

Sample Autocorrelations

Sample Auto-Correlation $r(k)$

$$r(k) = \frac{\sum_{t=k+1}^{T} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T} (y_t - \bar{y})^2}. \qquad \bar{y} := \frac{1}{T}\sum_{t=1}^{T} y_t$$

Population Auto-Correlation $\rho(k)$

$$\rho(k) = \frac{\mathbb{E}[(Y_t - \mathbb{E}[Y_t])(Y_{t-k} - \mathbb{E}[Y_{t-k}])}{\mathbb{E}[(Y_t - \mathbb{E}[Y_t])^2]}$$

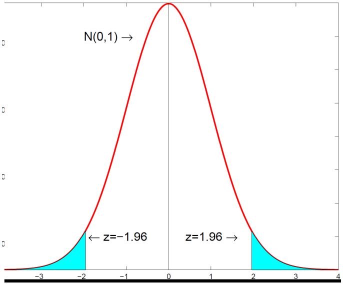
How to test $\rho(k) = 0$?

Autocorrelations: Statistical Properties

- Central Limit Theorem: If Y_t is a White Noise Process

$$\sqrt{T} r(k) \xrightarrow{d} N(0,1) \quad \text{all } k = 1, 2, \ldots$$

$$\Pr\left\{-1.96 \leq \sqrt{T} r(k) \leq 1.96\right\} \approx \Pr\{-1.96 \leq Z \leq 1.96\} = 0.95$$



Autocorrelations: Hypothesis Testing

$$H_0 : \ \rho(k) = 0$$
$$H_A : \ \rho(k) \neq 0$$

$$\boxed{\text{Under } H_0} \ \Pr\left\{-1.96 \leq \sqrt{T} r(k) \leq 1.96\right\} = 0.95$$

Rejection Region: $|\sqrt{T} r(k)| > 1.96$ evidence to reject H_0 at 5% level

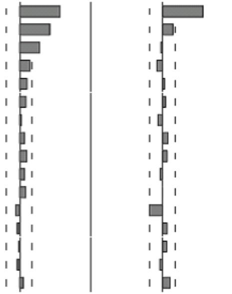
Acceptance Region: $|\sqrt{T} r(k)| \leq 1.96$ no evidence to reject H_0

Eviews Correlogram Output: User Guide

Correlogram of DLGDP

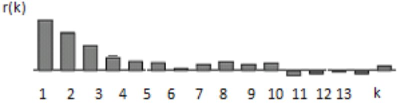
Sample: 1984Q1 2012Q4
Included observations: 116

Autocorrelation | Partial Correlation



	AC	PAC	Q-Stat	Prob
1	0.582	0.582	40.333	0.000
2	0.439	0.151	63.439	0.000
3	0.289	-0.024	73.582	0.000
4	0.151	-0.078	76.385	0.000
5	0.111	0.037	77.893	0.000
6	0.094	0.048	78.995	0.000
7	0.033	-0.060	79.129	0.000
8	0.075	0.080	79.836	0.000
9	0.104	0.073	81.214	0.000
10	0.078	-0.032	81.994	0.000
11	0.084	0.005	82.923	0.000
12	-0.051	-0.182	83.263	0.000
13	-0.033	0.067	83.406	0.000
14	-0.011	0.061	83.421	0.000
15	-0.032	-0.035	83.560	0.000
16	0.056	0.112	83.982	0.000

Properties of UK GDP Growth Example from Correlogram



1. Strong evidence of positive dependence in the series. Reject null $\rho(1), \rho(2), \rho(3)$ are zero as $r(1), r(2), r(3)$ in rejection region

2. Correlations positive and quickly decreasing, looks like AR(1) ACF

   Gives indication of form of true process

   True process may be AR(1), though could be ARMA(1,1) or AR(2)..

   Need method to estimate coefficients of true process..

AR(1) Estimation

Suppose $Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \text{WN}(\sigma^2)$

How to estimate μ, ϕ_1?

OLS- regress y_t(ind. variable) on a constant & y_{t-1} (dep. variable)

$$\hat{\alpha} = \bar{y} - \hat{\phi}_1 \bar{y}_{-1} \qquad \bar{y}_{-1} = \frac{1}{T-1} \sum_{t=2}^{T} y_{t-1}$$

$$\hat{\phi}_1 = \frac{\sum_{t=2}^{T} (y_t - \bar{y})(y_{t-1} - \bar{y}_{-1})}{\sum_{t=2}^{T} (y_{t-1} - \bar{y}_{-1})^2}$$

AR(1) Estimation: Properties

If $Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \text{WN}(\sigma^2)$ and $|\phi_1| < 1$

Consistency: $\hat{\phi}_1 \xrightarrow{p} \phi_1$

Central Limit Theorem: $\sqrt{T}(\hat{\phi}_1 - \phi_1) \xrightarrow{d} N\left(0, \omega^2\right)$

Use to form **hypothesis test on** ϕ_1

Result rely on ε_t **Serially Uncorrelated**

i.e ε_t uncorrelated with all past values

OLS Biased if errors **Serially Correlated**

AR(p) Estimation

$$Y_t = \mu + \phi_1 Y_{t-1} + .. + \phi_1 Y_{t-p} + \varepsilon_t$$

Regress y_t on constant $y_{t-1}, ..y_{t-p}$

$T - p$ observations for estimation

Consistency and CLT hold if ε_t is Serially Uncorrelated and the Process is stationary.

AR(p) Coefficient Testing

Joint Hypothesis Test on AR(p) Coefficients

$$
\begin{aligned}
H_0 &: \quad \phi_1 = ... = \phi_p = 0 \\
H_A &: \quad \text{at least one } \phi_i \neq 0,\ i = 1, ..., p
\end{aligned}
$$

- Run OLS, Perform F-Test of Joint Hypothesis
- t-test for individual test, e.g $\phi_1 = 0$

Test calculated and performed numerically in Eviews..

Testing for Serially Correlated Errors

$$Y_t = \mu + \phi_1 Y_{t-1} + .. + \phi_1 Y_{t-p} + \varepsilon_t$$

If ε_t serially correlated $=>$ ε_t correlated $Y_{t-1}, ..Y_{t-p}$ OLS BIASED

ε_t correlated $=>$ AR(p) does not explain all dynamics

Can test using Breusch Godfrey LM Test (see Video Lecture)

Test is calculated and performed in Eviews..

Estimating ARMA(p,q) Models

Estimation MA models slightly different to AR [See Notes]

Can estimate coefficients of $ARMA(p, q)$:
$Y_t = \mu + \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_t$

Can perform hypothesis tests on coefficients as for AR(p)

Also requires stationarity and serially uncorrelated errors.

Estimated in Eviews along with tests

Model Specification

- Need to decide appropriate lag orders for $ARMA(p, q)$

  Historically based on correlogram

  But often difficult to interpret & ambiguous

- Can estimate ARMA(p,q) and perform hypothesis tests on coefficients..

- Alternative approach based on **Information Criteria**

  Essentially measures to compare how well a model fits to the data

  Popular: **Akaike Information Criterion** & **Schwarz Information Criterion**

Information Criteria: Definition

- Estimate $ARMA\,(p, q)$:
  $Y_t = \mu + \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q}$
  $$\hat{\sigma}^2 = \frac{RSS}{T^*}$$

  Akaike Information Criterion

  $$AIC = \log \hat{\sigma}^2 + \frac{2\,(p+q+1)}{T^*}$$

  Schwarz Information Criterion

  $$SIC = \log \hat{\sigma}^2 + \frac{(p+q+1)\log T^*}{T^*}$$

- Trade-off goodness of fit ($\hat{\sigma}^2$, or RSS) against no. of coefficients $(p+q+1)$

  Increasing p and.or q reduces $\hat{\sigma}^2$

  Penalised by increase in second term

Using Information Criteria

- Estimate a range of models
  Say $ARMA(p, q)$ for all $0 \leqslant p \leqslant p_{\text{max}}$ and $0 \leqslant q \leqslant q_{\text{max}}$
  - In practice consider only a subset
- Select model delivering lowest value of relevant criterion
  - Automatic, but still requires specification of p_{max} & q_{max}
- Different criteria (eg AIC & SIC) can select different $ARMA$ models

Information Criteria Properties

$$
\begin{aligned}
AIC &= \log \hat{\sigma}^2 + \frac{2\,(p+q+1)}{T^*} \\
SIC &= \log \hat{\sigma}^2 + \frac{(p+q+1)\log T^*}{T^*}
\end{aligned}
$$

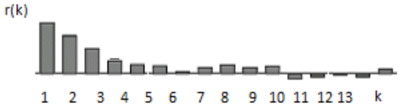
- SIC penalises more heavily $p+q+1$, unless T^* very small
- Consequently, for AR

  $$p_{AIC} \geq p_{SIC}$$

  & similarly for MA
- For true $AR(p_0)$ then provided $p_{\max} \geq p_0$

  $$
  \begin{aligned}
  \text{as } T^* &\rightarrow \infty,\ p_{SIC} \rightarrow p_0 \\
  \text{as } T^* &\rightarrow \infty,\ p_{AIC} \geq p_0
  \end{aligned}
  $$

Example: UK GDP Growth



Evidence of AR(1)

Possibly MA(2) or MA(3)

Estimation & Information Criterion to Select model using Eviews.

Eviews:MA(2) Estimation

$$MA(2) \quad Y_t = \alpha + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t$$

Dependent Variable: DLGDP
Method: Least Squares
Sample: 1984Q1 2012Q4
Included observations: 116
Convergence achieved after 17 iterations
MA Backcast: 1983Q3 1983Q4

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.599462	0.097589	6.142706	0.0000
MA(1)	0.432890	0.088259	4.904768	0.0000
MA(2)	0.343070	0.089958	3.813683	0.0002

R-squared	0.314704	Mean dependent var	0.601666
Adjusted R-squared	0.302575	S.D. dependent var	0.710753
S.E. of regression	0.593564	Akaike info criterion	1.820178
Sum squared resid	39.81196	Schwarz criterion	1.891392
Log likelihood	-102.5703	Hannan-Quinn criter.	1.849087
F-statistic	25.94618	Durbin-Watson stat	1.796944
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.831882	Prob. F(4,109)	0.0280
Obs*R-squared	10.91930	Prob. Chi-Square(4)	0.0275

Eviews: AR(1) Estimation

$$AR(1) \quad Y_t = \alpha + \phi_1 Y_{t-1} + \varepsilon_t$$

Dependent Variable: DLGDP
Method: Least Squares
Sample: 1984Q1 2012Q4
Included observations: 116

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.238187	0.071080	3.350964	0.0011
DLGDP(-1)	0.591083	0.075976	7.779879	0.0000

R-squared	0.346804	Mean dependent var	0.601666
Adjusted R-squared	0.341074	S.D. dependent var	0.710753
S.E. of regression	0.576949	Akaike info criterion	1.754964
Sum squared resid	37.94714	Schwarz criterion	1.802439
Log likelihood	-99.78790	Hannan-Quinn criter.	1.774236
F-statistic	60.52652	Durbin-Watson stat	2.106444
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.639092	Prob. F(4,110)	0.6357
Obs*R-squared	2.634579	Prob. Chi-Square(4)	0.6207

Recap

Problem: How to infer form of true process from data?

1. Sample Correlations & Correlogram
2. Estimation of ARMA Models & testing
3. Serial correlation testing (see video)
4. Information Criterion (AIC/SIC)
5. Eviews example: UK GDP Growth

Next Week..

Prediction from ARMA Models

Seasonality in Time Series

Ex. Class on Estimation/Testing **(Wk4)** & PC Lab in Eviews **(Wk 5)**

....Ends my part of the course....

Move to **Multivariate Time Series Analysis** with Prof. Alastair Hall