# ECON30401: Time Series Econometrics

Lecture 1: Introduction to Time Series Econometrics

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# Course Details

# Some Background Information

Time Series Econometrics- tool to analyse economic time series data

*Particularly macroeconomic and financial data*

Builds on time series methods in ECON20110 Basic Econometrics

*Prerequisite for ECON31012 Financial Econometrics*

Mixture of theoretical & empirical methods

*Will use EViews software*

Nicky Grant (wk. 1-4) - Prof. Alastair Hall (wk 6-10)

**Nicky**: Mon 2-3pm & Fri 1-2pm (ALB 2.007)

**Alastair**: Tues: 3-4pm & Fri 1-2pm.

# Syllabus Overview

Course roughly split in to 6 parts over 8 lectures.

1. Introduction to Time Series
2. Stationarity and stationary processes: ($AR$, $MA$, $ARMA$)
3. Estimation, Hypothesis Testing & Model Selection
4. Prediction & Seasonality

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5. System modelling: Vector autoregressive ($VAR$) models

   *(Extends 1-4 for multivariate models (not inc. seasonality))*
6. Random Walks & Unit roots.

# Administrative Arrangements

Slides available on Blackboard in advance of each lecture

Lecture notes covering the topics in lecture

- Text books also help understanding
- Enders *Applied Econometric Time Series* recommended

Classes start week 3 (PC Labs and exercises).

- Attempt exercise questions before class **(Extremely Important)**
- Including applied work on EViews, using real data (provided)

# Assessment and Support Hours

Assessment:

**20% empirical project**

Two weeks to complete- submit last week of term.

Will use EViews to analyse data

**80% final examination in January**

1.5 hours duration

2 support sessions in last weeks of term (2 hours each)

Not compulsory

Mock & past exam papers on Blackboard before Christmas Break.

# Introduction to Time Series

# Motivation

How to infer properties of a variable(s) observed over time?

e.g Interest Rates, Unemployment, Stock Prices.

Model properties of time series **esp. dependence over time**

- future interest rates on past interest rates (univariate)
- future inflation on past interest rates (multivariate)

Differences with Cross Section:

- Temporal ordering
- Potential dependence between observations

# Time Series Process

**Time Series Process** $Y_t$ $(t = .., -1, 0, 1, ...)$

[$Y_t$ *is a Random Variable*]

Observe a **Realisation** $\{y_t\}_{t=1}^{T}$

[*Data Sample of size* $T$]

Sample realisations from $Y_t = 0.5 + 0.4Y_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0, 1)$

Aim: Develop methods to infer properties of the true **Process** using **Sample Realisation**.

# Time Series vs. Sample Realisation: Empirical Example

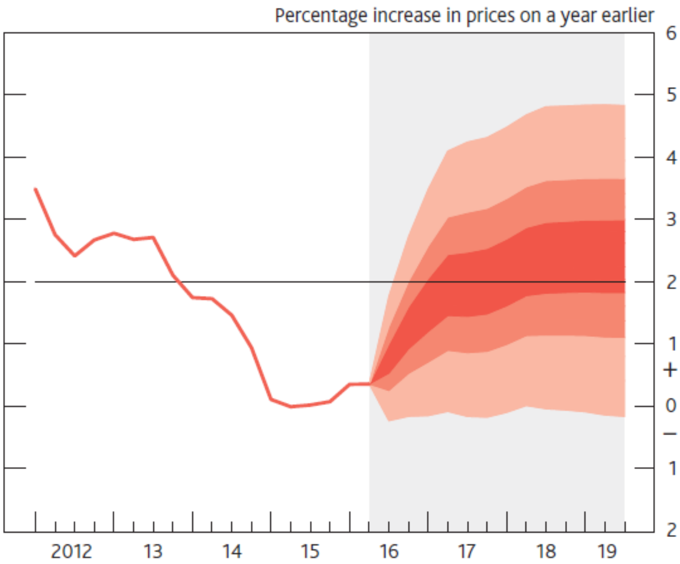



**Figure 1:** UK CPI and Projections Based on Market Expectations in August 2016

Outturns of inflation are expected to lie within each pair of the lighter red areas on 30 occasions. In any particular quarter of the forecast period, inflation is therefore expected to lie somewhere within the fans on 90 out of 100 occasions. On the remaining 10 out of 100 occasions inflation can fall anywhere outside the red area

Source: Bank of England Inflation Report, August 2016, pg. 39

# Sample Realisations of Time Series Processes

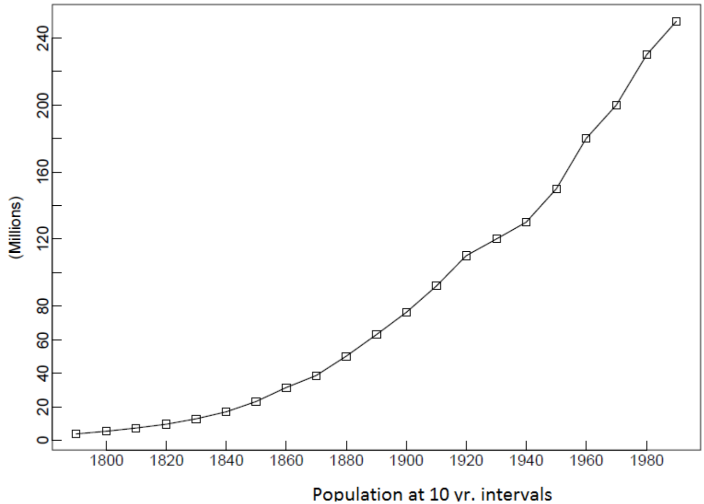



**Figure 2:** US Population at Ten Year Intervals

Source: Introduction to Time Series (Brockwell), pg. 5

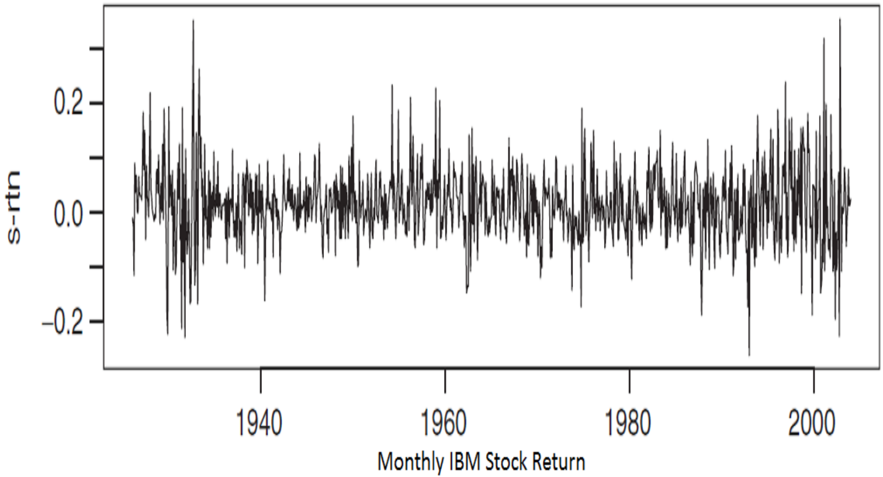



**Figure 3:** Monthly IBM Stock Return

Source: Analysis of Financial Time Series (Tsay), pg.18

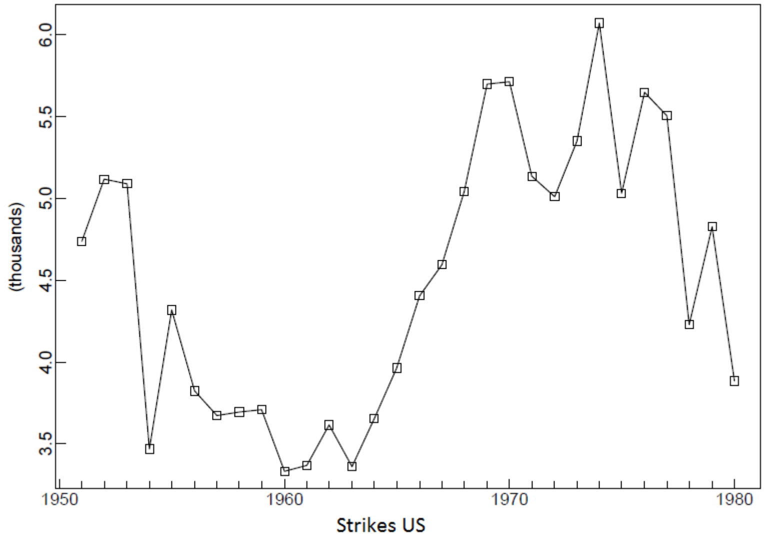



**Figure 4:** Total Yearly US Strikes

SOURCE: INTRODUCTION TO TIME SERIES (BROCKWELL), PG.5

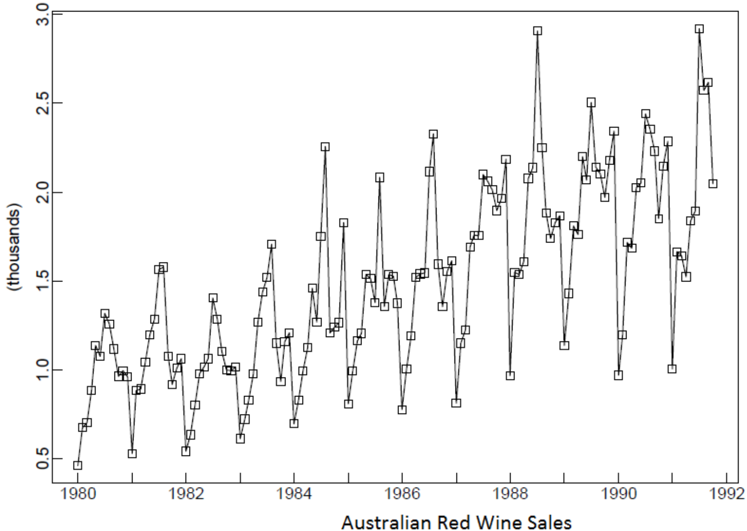



**Figure 5:** Monthly Australian Red Wine Sales

Source: Introduction to Time Series (Brockwell), pg. 2

# Quick Stats Recap

# Stats Recap: Moments of Distributions

**Definition: Expectation Operator**

For any r.v's $X$,$Y$ and any function $g(\cdot,\cdot)$ the operator $\mathbb{E}[\cdot]$ satisfies

$$\mathbb{E}[g(X,Y)] := \sum_x \sum_y g(x,y) \Pr\{X = x, Y = y\}$$ **DISCRETE**

$$\mathbb{E}[g(X,Y)] := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y) dx dy$$ **CONTINUOUS**

Example: Moments of a Random Variable

$\mathbb{E}[X]$ Mean of X

$\mathbb{E}[(Y - E[Y])^2]$ Variance of Y

$\mathbb{E}\left[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])\right]$ Covariance of X,Y

# Key Definitions

# Stationarity

**Definition: Stationary Process**

A process $Y_t$ is **Stationary** if

1. $\mathbb{E}[Y_t] = \mu$ for all $t$
2. $Var[Y_t] = \sigma^2 < \infty$ for all $t$
3. $Cov[Y_t, Y_{t-j}] = \gamma(|j|)$ for each $j$ & for all $t$

Time Series Process has desirable properties if is Stationary

*The assumption may not hold in practise*

1. Variance of $Y_t$ changes over time
2. Mean of $Y_t$ changes over time
3. $Y_t$ has a unit root (e.g a Random Walk) [Lecture 8]

# White Noise Process

**Definition: White Noise**

A process $Y_t$ is **White Noise** if

1. $\mathbb{E}[Y_t] = 0$ for all $t$
2. $\mathbb{E}[Y_t^2] = \sigma^2 < \infty$ for all $t$
3. $\mathbb{E}[Y_t Y_{t-j}] = 0$ for all $t$ & for any $j \neq 0$

Shorthand notation $Y_t \sim$WN$(\sigma^2)$

**Note that:** White Noise $\Rrightarrow$ Stationarity, but not the reverse

# Some Simple Univariate Time Series Models

**Autoregressive**: depends on own past values

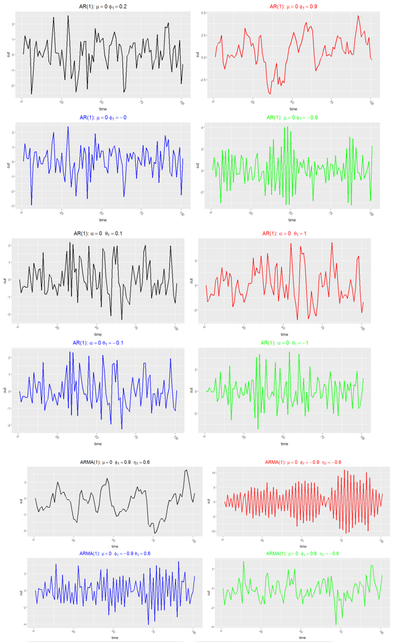
AR(1) : $Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim WN(\sigma^2)$

**Moving Average** : weighted average of recent 'shocks'

MA(1): $Y_t = \alpha + \varepsilon_t + \theta_1 \varepsilon_{t-1} \qquad \varepsilon_t \sim WN(\sigma^2)$

**Autoregressive Moving Average**: AR(1) & MA(1) Component

ARMA(1,1): $Y_t = \mu + \phi_1 Y_{t-1} + \varepsilon_t + \eta_1 \varepsilon_{t-1} \quad \varepsilon_t \sim WN(\sigma^2)$



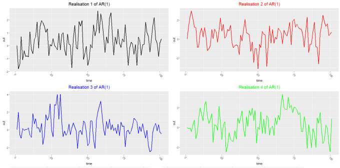
# Fundamental Aim of Time Series

Suppose true unobservable process is $Y_t = 0.1 + 0.4Y_{t-1} + \epsilon_t$ where $\varepsilon_t \sim WN(\sigma^2)$

Since $\{y_t\}_{t=1}^T$ drawn from $Y_t$ - *should reflect properties of the process (under some assumptions)*

How to correctly infer properties of $Y_t$ **given sample realisation** $\{y_t\}_{t=1}^T$?

Difficult to tell from the plotting the graph alone




Require framework to succinctly describe properties of a process...

# Auto-covariance & Autocorrelation Function

**Definition: Autocovariance Function of $Y_t$**

$$\gamma(k) := Cov(Y_t, Y_{t-k}) \qquad k = 0, 1, 2, ...$$

**Definition: Autocorrelation Function of $Y_t$**

$$\rho(k) := \frac{Cov(Y_t, Y_{t-k})}{\sqrt{Var(Y_t)Var(Y_{t-k})}} = \frac{\gamma(k)}{\gamma(0)} \qquad k = 0, 1, 2, ...$$

$\rho(k)$ for $k = 1, 2, ..$ reflect dependence properties of $Y_t$

We can derive $\rho(k)$ for different forms of processes

Correlation in sample data should reflect the correlation in $Y_t$

Can use this idea to infer form of $Y_t$ (Lecture 3 & 4)

# Theoretical Properties of MA Process

# MA(1) Process: Mean & Variance

MA(1): $\quad Y_t = \alpha + \varepsilon_t + \theta_1 \varepsilon_{t-1}, \quad \varepsilon_t \sim WN(\sigma^2)$

## Mean of MA(1)

$$
\begin{aligned}
\mathbb{E}\left[Y_t\right] &= \mathbb{E}[\alpha + \varepsilon_t + \theta_1 \varepsilon_{t-1}] \\
&= \alpha + \theta_1 \mathbb{E}[\varepsilon_{t-1}] + \mathbb{E}[\varepsilon_t] \\
&= \alpha
\end{aligned}
$$

## Variance of MA(1)

$$
\begin{aligned}
\mathrm{Var}\left[Y_t\right] &= \mathbb{E}[(Y_t - E[Y_t])^2] \\
&= \mathbb{E}[(Y_t - \alpha)^2] \\
&= \mathbb{E}\left[(\varepsilon_t + \theta_1 \varepsilon_{t-1})^2\right] \\
&= \mathbb{E}[\varepsilon_t^2] + \theta_1^2 \mathbb{E}[\varepsilon_{t-1}^2] + 2\theta_1 \mathbb{E}[\varepsilon_t \varepsilon_{t-1}] \\
&= \sigma^2 (1 + \theta_1^2)
\end{aligned}
$$

# MA(1) Process: Autocovariance Function

Autocovariance Function of MA(1)

$$
\begin{aligned}
\operatorname{Cov}[Y_t, Y_{t-1}] &= \mathbb{E}\left[\left(Y_t - \mathbb{E}[Y_t]\right)\left(Y_{t-1} - \mathbb{E}[Y_{t-1}]\right)\right] \\
&= \mathbb{E}\left[\left(Y_t - \alpha\right)\left(Y_{t-1} - \alpha\right)\right] \\
&= \mathbb{E}\left[\left(\varepsilon_t + \theta_1\varepsilon_{t-1}\right)\left(\varepsilon_{t-1} + \theta_1\varepsilon_{t-2}\right)\right] \\
&= \mathbb{E}[\varepsilon_t\varepsilon_{t-1} + \theta_1\varepsilon_t\varepsilon_{t-2} + \theta_1\varepsilon_{t-1}^2 + \theta_1^2\varepsilon_{t-1}\varepsilon_{t-2}] \\
&= \mathbb{E}[\varepsilon_t\varepsilon_{t-1}] + \theta_1\mathbb{E}[\varepsilon_t\varepsilon_{t-2}] + \theta_1\mathbb{E}[\varepsilon_{t-1}^2] + \theta_1^2\mathbb{E}[\varepsilon_{t-1}\varepsilon_{t-2}] \\
\\
&= \theta_1\mathbb{E}[\varepsilon_{t-1}^2] \\
&= \theta_1\sigma^2
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Cov}(Y_t, Y_{t-2}) &= \mathbb{E}\left[(\varepsilon_t + \theta_1\varepsilon_{t-1})(\varepsilon_{t-2} + \theta_1\varepsilon_{t-3})\right] \\
&= 0
\end{aligned}
$$

$$
Cov(Y_t, Y_{t-j}) = 0 \qquad\qquad j \geq 2
$$

# MA(q) Process

**Definition: Moving Average Process of Order q (MA(q))**

$$Y_t = \alpha + \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} \qquad \varepsilon_t \sim \mathrm{WN}(\sigma^2)$$

for some coefficients $\alpha, \theta_1, ..\theta_q$ where $\theta_0 = 1$ (normalisation).

**Mean of MA(q)**: $\mathbb{E}\left[Y_t\right] = \alpha$

**Variance of MA(q)**: $Var\left[Y_t\right] = \sigma^2 \left(1 + \theta_1^2 + ... + \theta_q^2\right)$

**Autocovariance Function of MA(q)**: $\gamma(k) = \begin{cases} \sigma^2 \sum_{s=0}^{q-k} \theta_s \theta_{s+k} & : k = 0, ., q \\ 0 & : k > q \end{cases}$

# Recap

# Lecture Recap

Distinction between true (unobserved) **process** $Y_t$ and sample **realisation** (data) $y_t$

Plotted realisation of some time series variables

Key Definitions & Concepts- **Stationarity** & **Autocovariance/correlation Functions**

Introduced Simple Univariate Time Series Processes **(MA, AR, ARMA)**

Theoretical **Properties of MA(q)** Models (mean,variance and ACFs)

Video clips available in these slides, lectures notes & Youtube Channel

# Next Week

## Next Week

Introduce $MA(\infty)$ and discuss its importance

Properties of AR(1), ARMA(1,1) and introduce general ARMA(p,q) process

In particular discuss Autocorrelation/covariance Function of AR(1), ARMA(1,1)

Conditions for stationarity in ARMA models

**Reading:** **Chapter 1** (this weeks material) and read **Chapter 2** in preparation for next week.