

Exercise Sheet 3

1. Consider the following univariate *ARMA* model for quarterly seasonal data

$$y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \phi_4 y_{t-4} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \quad (1)$$

where  $\varepsilon_t$  is white noise with variance  $\sigma^2$ . Suppose that all coefficients of (1) are known, and that values  $y_1, \dots, y_t$  have been observed, together with the true disturbances  $\varepsilon_1, \dots, \varepsilon_t$ . Finally, assume that the latest available observation  $y_t$  relates to the fourth quarter of the year.

- (a) Write down recursive equations that could be used to predict the values  $y_{t+i}$ , for  $i = 1, 2, 3, 4, 5$ .  
 (b) Analogous to Exercise Sheet 1, the mean-square error for an  $i$ -step ahead prediction is defined as

$$MSE(i) = var[y_{t+i} | y_t, y_{t-1}, \dots, \varepsilon_t, \varepsilon_{t-1}, \dots] = E[y_{t+i} - \hat{y}_{t+i}]^2$$

where  $\hat{y}_{t+i} = E[y_{t+i} | y_t, y_{t-1}, \dots]$ . Find expressions for  $MSE(1)$  and  $MSE(2)$  for the seasonal process of (1).

- (c) For this process, will  $MSE(3)$  and  $MSE(4)$  differ from  $MSE(2)$ ? What about  $MSE(5)$ ? [You are not required to obtain expressions for the  $MSE$  to answer this part.]  
 (d) How will the prediction equations in part (a) be modified if sample observations on  $y_t$  are available, but the true coefficients and disturbances of (1) are not known?

2. Consider a bivariate (ie, two variable) *VAR(1)* system

$$\mathbf{y}_t = \alpha + \Phi_1 \mathbf{y}_{t-1} + \varepsilon_t$$

where  $\varepsilon_t$  is vector white noise.

- (a) Derive the *VMA*( $\infty$ ) representation of  $\mathbf{y}_t$  stating the condition required on  $\Phi^j$  as  $j \rightarrow \infty$  in order the for process to be stationary.  
 (b) If you are also given that

$$\alpha = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},$$

find  $E[\mathbf{y}_t]$  for each matrix  $\Phi_1$  in (a) that corresponds to a stationary process.

3. Let  $v_t$  be the  $2 \times 1$  vector,

$$\mathbf{v}_t = \mathbf{e}_t + \Theta_1 \mathbf{e}_{t-1}$$

where  $\mathbf{e}_t = (e_{1,t}, e_{2,t})' \sim i.i.d.(0, \Sigma)$ ,

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_2^2 \end{bmatrix},$$

and

$$\Theta_1 = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} \\ \theta_{2,1} & \theta_{2,2} \end{bmatrix}.$$

Define  $\Gamma_1 = E[v_t v_{t-1}']$  and  $\Gamma_{-1} = E[v_t v_{t+1}']$ . Show that  $\Gamma_1 \neq \Gamma_{-1}$  but  $\Gamma_1 = \Gamma_{-1}'$ .