Financial Econometrics

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EC5609: Problem Set 1

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1. For the standard uniform random variable $X \sim Unif[0,1]$ where the probability density function is $f(x) = 1$ for $0 \leq x \leq 1$ for 0 otherwise calculate the mean, variable, skewness and kurtosis.

2. For two random variables X and Y, provide a proof of the Law of Iterated Expectations, namely that

$$E_X[X] = E_X\left[E_{Y|X}[Y|X]\right] \tag{1}$$

you may assume that X,Y are continuously distributed (likewise the proof can be shown for discrete variables).

3. For $\mathbf{X} = (x_1, \cdots x_T)'$ where x_t are $k \times 1$ vectors for $j = \{1, \cdots T\}$ and $\mathbf{y} = (y_1, \cdots y_T))'$ show that $\mathbf{X}'\mathbf{y} = \sum_{t=1}^T x_t y_t$ and $\mathbf{X}'\mathbf{X} = \sum_{t=1}^T x_t x_t'$

4. Consider the linear model $y_t = x_t'\beta_0 + u_t$ for $t = 1, \cdots, T$ where u_t, x_t satisfy assumptions A1-A6 in Lecture 3. (In this question all elements are defined as in Lecture 3).
   (a) Show that the OLS estimate $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$
   (b) Show that the OLS estimator is unbiased and explain which assumptions you use to show that $E[\hat{\beta}] = \beta_0$.
   (c) Show that $Var(\hat{\beta}|\mathbf{X}) = \sigma_u^2(\mathbf{X}'\mathbf{X})^{-1}$ where $\sigma_u^2 = E[u_t^2|\mathbf{X}]$.
   (d) If errors are not spherical, briefly explain why forming paramater restriction tests estimating OLS standard errors based on the formula in (c) will generally be biased.

5. The weekly returns of the Apple stock for 1/1/01 to 12/2/01 are (0.101,0.050, 0.135, 0.003,0.0543,-0.072, -0.007, -0.007,-0.009).
   Using this data set of size $T = 9$ answer the following (all the below can be done on a calculator, and I would advise you use a calculator in this example).
   (a) Calculate the sample mean and variance.
   (b) Calculate the sample autocorrelation at lag 1 and 2.
   (c) Perform a 95% two sided hypothesis test that the population correlation at lag 1 is zero (state all the steps, the test statistic and distribution and any assumptions made).
   (d) Perform a 95% two sided joint hypothesis test that the population correlation at lag 1 and 2 are both zero (state all the steps, the test statistic and distribution and any assumptions made).
   (e) Why may we be sceptical of inference from the results in (c),(d).

6. Let $\mathcal{F}_t$ denote the information set of all past prices and $\mathcal{G}_t$ the information set with all past prices and all public information.
   (a) Provide a statement of the weak and semi strong form of the efficient market hypothesis (EMH) and explain this result.

(b) Show that the semi strong form of EMH implies the weak form.

(c) Show that the weak form EMH implies returns in different time periods are uncorrelated when the normal rate of return is constant.

(d) If normal rate of return is not constant, does the result in (c) hold, and if not explain why.

7. Below is the joint distribution of returns R_t for $t = 1$ and $t = 2$ that take 3 values, 0,1 or 2 percent.

(a) Derive the marginal distributions of R_1 and R_2.

(b) Derive the mean and variance of R_1, R_2.

(c) Find $Cov(R_1, R_2)$

(d) Find the conditional distribution of R_1 given $R_2 = 0, 1$ or 2 respectively.

(e) What does (d) tell us about the validity of the efficient market hypothesis?