Financial Econometrics [EC5609]

Lect 7:Advanced Volality Modelling

Nicky Grant (Semester 1, 2021/2022)

Volatility Modelling- (Generalised) Autoregressive Heteroscedasticity

Conditional Heteroskedasticity

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Can test and select between using AIC/BIC and parameter restriction tests similar to ARMA modelling

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All these we can extend using STATA software

Asymmetric Volatility Effect

(i) **Threshold GARCH (TARCH)**
The TARCH(1,1) specification (Glosten, Jagannathan, and Runkle, 1993; Zakoïan, 1994) of the conditional variance is

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} + \lambda u_{t-1}^2 I_{t-1},$$

where I_{t-1} is an indicator variable defined as

$$I_{t-1} = \begin{cases} 1 : u_{t-1} \geq 0 \\ 0 : u_{t-1} < 0. \end{cases}$$

To make the asymmetry in the effect of news on the conditional variance explicit, this model can be written as

$$h_t = \begin{cases} \alpha_0 + (\alpha_1 + \lambda) u_{t-1}^2 + \beta_1 h_{t-1} & : u_{t-1} \geq 0 \\ \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} & : u_{t-1} < 0. \end{cases}$$

The leverage effect in equity markets would lead us to expect $\lambda < 0$, so that negative news, with $u_{t-1} < 0$, is associated with a higher effect on volatility than positive news of the same magnitude.

(ii) **Exponential GARCH (EGARCH)**
The EGARCH(1,1) specification (Nelson, 1991) of the conditional variance is

$$\log h_t = \alpha_0 + \alpha_1 \left| \frac{u_{t-1}}{\sqrt{h_{t-1}}} \right| + \lambda_1 \frac{u_{t-1}}{\sqrt{h_{t-1}}} + \beta_1 \log h_{t-1}.$$

An important advantage of the EGARCH specification is that the conditional variance is guaranteed to be positive at each point in time. This result follows from the fact that the variance is expressed in terms of $\log h_t$, so the actual variance is obtained by exponentiation. The parameter α_1 captures potential asymmetry in the effect of u_{t-1} on $\log h_t$. It is expected that $\alpha_1 < 0$, so negative news is associated with a higher effect than positive news of the same magnitude.

... are fitted to daily returns from

GARCH(1,1) with generalised error distribution

ARCH family regression -- ARMA disturbances

Sample: 1/31/2000 thru 10/25/2021 | Number of obs = 1135
Wald chi2(2) = 48.20
Log likelihood = 1697.396 | Prob > chi2 = 0.0000

Amzn	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Amzn						
_cons	.0048889	.0010842	4.51	0.000	.0027639	.0070139
ARMA						
ar						
L1.	.8030736	.197709	4.06	0.000	.415571	1.190576
ma						
L1.	-.8299151	.1846082	-4.50	0.000	-1.191741	-.4680896
ARCH						
arch						
L1.	.0392042	.0096936	4.04	0.000	.0202051	.0582033
garch						
L1.	.9525773	.0106308	89.61	0.000	.9317414	.9734131
_cons	.0000225	.0000116	1.94	0.052	-1.96e-07	.0000453
/lnshape	.186914	.0421016	4.44	0.000	.1043965	.2694316
shape	1.205524	.0507544			1.11004	1.30922

Figure 1: GARCH(1) GED Model: Amazon Weekly Stock Return 2000-2021

GARCH(1,1) in mean Example

ARCH family regression -- AR disturbances

Sample: 1/31/2000 thru 10/25/2021 — Number of obs = 1135

Wald chi2(2) = 1.75

Log likelihood = 1639.353 — Prob > chi2 = 0.4176

Amzn	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Amzn						
_cons	.0057448	.0021006	2.73	0.006	.0016278	.0098618
ARCHM						
sigma2	.4356567	.584818	0.74	0.456	-.7105656	1.581879
ARMA						
ar L1.	-.0273474	.0322849	-0.85	0.397	-.0906245	.0359298
ARCH						
arch L1.	.040234	.0052499	7.66	0.000	.0299445	.0505235
garch L1.	.9543409	.0050358	189.51	0.000	.9444708	.9642109
_cons	.0000186	6.03e-06	3.09	0.002	6.80e-06	.0000304

Figure 2: GARCH in Mean Model: Amazon Weekly Stock Return 2000-2021

Nelsons Exponential GARCH (EGARCH)

ARCH family regression -- MA disturbances

Sample: 1/31/2000 thru 10/25/2021 Number of obs = 1135
Wald chi2(1) = 0.56
Log likelihood = 1655.092 Prob > chi2 = 0.4551

Amzn	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Amzn						
_cons	.0054849	.0014477	3.79	0.000	.0026474	.0083224
ARMA						
ma L1.	-.0221692	.0296778	-0.75	0.455	-.0803367	.0359982
ARCH						
earch L1.	-.0610696	.0082118	-7.44	0.000	-.0771645	-.0449747
earch_a L1.	.0572982	.0081002	7.07	0.000	.0414221	.0731743
egarch L1.	.9950414	.0015257	652.21	0.000	.9920512	.9980317
_cons	-.0261635	.0078858	-3.32	0.001	-.0416195	-.0107076

Figure 3: GARCH(1,3) Model: Apple Stock Return 2000-2021

Threshold GARCH (TARCH)

ARCH family regression -- MA disturbances

Sample: 1/31/2000 thru 10/25/2021	Number of obs = 1135
	Wald chi2(1) = 0.55
Log likelihood = 1654.121	Prob > chi2 = 0.4601

Amzn	Coefficient	OPG std. err.	z	P>|z|	[95% conf. interval]	
Amzn						
_cons	.0054765	.0014651	3.74	0.000	.0026049	.0083481
ARMA						
ma						
L1.	-.0219678	.0297384	-0.74	0.460	-.0802539	.0363183
ARCH						
abarch						
L1.	.0631146	.0057298	11.02	0.000	.0518845	.0743447
atarch						
L1.	-.0598504	.0086708	-6.90	0.000	-.0768449	-.0428559
sdgarch						
L1.	.9696657	.0035755	271.20	0.000	.9626578	.9766735
_cons	.000313	.0000963	3.25	0.001	.0001242	.0005018

Figure 4: Threshold GARCH Model: Apple Stock Return 2000-2021