Financial Econometrics|EC5609

Wk1: Distribution Theory and Properties of Stock Price Returns I

Nicky Grant (Semester 1, 2021/2022)

Table of contents

Course Details

Module Information

Financial econometrics- branch of econometrics specifically focused on dealing with the challenges of financial economic data

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Office Hours: Online Monday 15:10-17:00 (or by appointment)

Lecture Topics

1 Distribution Theory and Properties of Stock Price Returns I

2 Distribution Theory and Properties of Stock Price Returns II

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8 Advanced Volatility Modelling

9 Unit Roots & Co-integration

Assessment and Reading

30% Empirical Project & 20% Class Test

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50% Final Examination

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Main Textbooks:

Hurn and Martin: Financial Econometric Modeling (**HM**)

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Supplementary Textbooks:

Linton: Financial Econometrics: models and methods (**L**)

Financial Econometric Data Examples

Stock Price Returns

Large part of the literature and EC5609 focuses on **stock price returns**

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See **HM** Chapter 1 for more general definitions taking into account **dividends** and also **continuously compounded** and **k-period ahead returns**

Example Financial Return Data

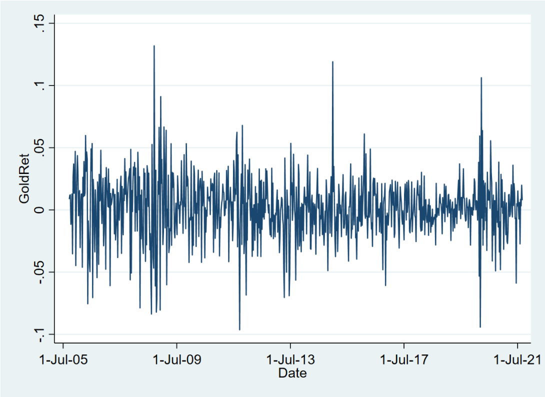

Figure 1: Weekly Gold Price Return 2005-2021

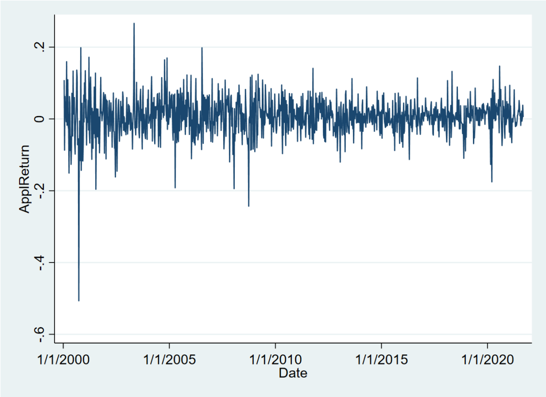

Figure 2: Weekly Apple Stock Return 2000-2021

Example Financial Return Empirical Densities

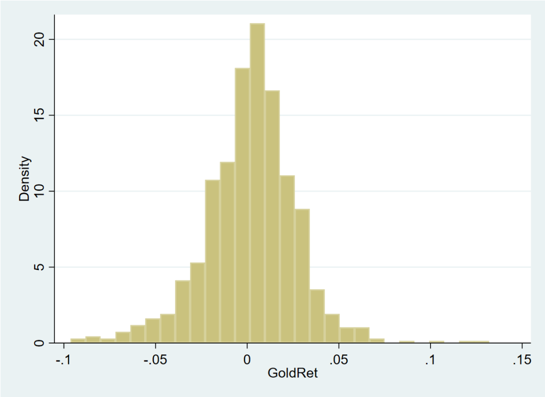

Figure 3: Empirical Density Weekly Gold Return 2005-2021

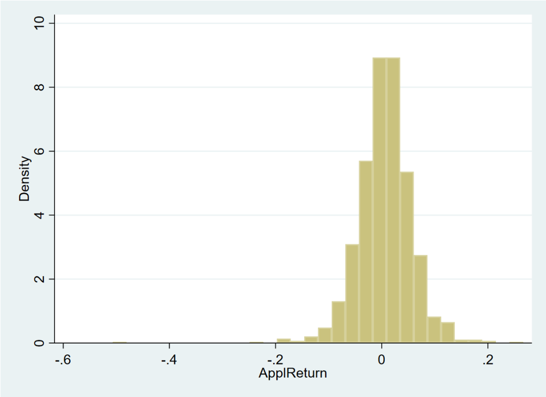

Figure 4: Empirical Density Weekly Apple Stock Return 2000-2021

Financial Asset Return Empirical Stylised Facts

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Slow decay in correlation is absolute returns

Statistical Distribution Theory and Definitions

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Often interested in modelling the distribution of financial economic data

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Crucial to understand key definitions and results on statistical distribution theory and sampling

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X **random variable** (r.v.) taking discrete values

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Satisfies axioms of probability

Discrete Variables: Example- Bernoulli

Takes two values: $\mathrm{Supp}(X) = \{0, 1\}$

$$f_X(x) = \begin{cases} p, & x = 1, \\ 1 - p, & x = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Continuous Variables: Univariate Probability Density and Distribution Functions

A continuous r.v. X takes values over a continuous interval, e.g. $\mathrm{Supp}(X) = \mathbb{R}$

Probability distribution function: $F_X(x) := \int_{-\infty}^{x} f_X(x)dx$

$f_X(x)$ is the **probability density function** at $X = x$

Continuous Variables: Example- Standard Uniform Distribution Function

Constant density function with Supp$(X) = [0, 1]$

$$f_X(x) = \begin{cases} 1, & 0 \geq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Continuous Variables: Example- Standard Distribution Function

Bell-shaped density function with $\mathrm{Supp}(X) = \mathbb{R}$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$$

Moments of a univariate (continuous) random variable

Definition: Population Moments of X

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$$\mathbb{E}[g(X)] := \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

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Var of X $\quad \mathbb{E}[(X - \mu_X)^2]$ $\quad \sigma_X^2 := \int_{-\infty}^{\infty} (X - \mu_X)^2 f_X(x) dx$

Higher order moments: Skewness and Kurtosis

Skewness of X: $\mathbb{E}\left[\left(\frac{X-\mu_x}{\sigma_X}\right)^3\right]$

Kurtosis of X: $\mathbb{E}\left[\left(\frac{X-\mu_x}{\sigma_X}\right)^4\right]$

Multivariate Random Variables

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We give definition for bivariate r.v.s $[k = 2]$ (but definition generalises- see)

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Marginal density $f_{X_1}(x_1) = \sum_{x_2} f_X(x_1, x_2)$

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$$\text{Cov}(X_1, X_2) := \mathbb{E}[(X_1 - \mu_{X_1})(X_2 - \mu_{X_2})] \quad \textbf{Covariance } X_1, X_2$$

$$\text{Corr}(X_1, X_2) := \text{Cov}(X_1, X_2)/\sigma_{X_1}\sigma_{X_2} \quad \textbf{Correlation } X_1, X_2$$

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Follows from Bayes Rule

$f_{X_1|X_2=x_2}(x_1)$ is a probability function- can use to work out probability statements on X_1, conditioning on the event $X_2 = x_2$

Conditional Moments

Sampling

Distribution of sample data: $\mathcal{X} = \{X_1, \cdots, X_T\}$

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$(x_1, \cdots, x_T)'$ is observed sample data from $\{X_1, \cdots, X_T\}$

Sampling and Moment Estimation

Sampling: i.i.d assumption

Would like to use observed data to estimate the distribution(or moments) generating the data

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This assumption often made with cross section data

Generally does not hold with time series data, where modelling the dependence is the main interest

Definition: Weakly Stationary Process

A process $\{Z_t : t \in \mathcal{T}\}$ is **weakly stationary** if

$$\mathbb{E}[Z_t] = \mu \quad \text{for all } t \in \mathcal{T},$$

$$\mathrm{Var}(Z_t) = \sigma^2 < \infty \quad \text{for all } t \in \mathcal{T},$$

$$\mathrm{Cov}(Z_{t_1}, Z_{t_2}) = \gamma(|t_1 - t_2|) \quad \text{for all } t_1, t_2 \in \mathcal{T}.$$

Stationarity Assumption

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Definition: Strictly Stationary Process

A process $\{Z_t\}$ is **strictly stationary** if for all $t_1, \cdots t_k$ in $\mathcal{T}$ the joint density of $(Z_{t_1+\tau}, ..., Z_{t_k+\tau})'$ does not depend on τ.

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Process generating sample data may not be stationary, e.g.

- Mean or (co-) variance of Z_t is not constant over time
- Z_t has a unit root (e.g g a random walk)

Definition: White Noise

A process $\{Z_t\}$ is **white noise** if

$$\begin{aligned} \mathbb{E}[Z_t] &= 0 && \text{for all } t \in \mathcal{T}, \\ \mathbb{E}[Z_t^2] &= \sigma^2 < \infty && \text{for all } t \in \mathcal{T}, \\ \mathbb{E}[Z_{t_1} Z_{t_2}] &= 0 && \text{for all } t_1 \neq t_2 \in \mathcal{T}. \end{aligned}$$

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Note: white noise $\Rrightarrow$ weak stationarity (not the reverse)

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Correlations in sample data should reflect the correlation in X_t (for large T)

Sample Estimation

Assuming DGP is (weakly) stationary, under some mild conditions, sample averages of moments consistently estimate the population moment.

Sample Mean: $\mathbf{X}_{\mathcal{T}} = \frac{1}{T}\sum_{T=1}^{T} X_t$ consistently estimates μ_X

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Assuming DGP is (weakly) stationary, under some mild conditions, sample averages of moments consistently estimate the population moment.

Sample Mean: $\mathbf{X}_{\mathcal{T}} = \dfrac{1}{T}\displaystyle\sum_{T=1}^{T} X_t$ consistently estimates μ_X

Likewise can construct time averages of variance, skenwness and kurtosis

Empirical Example Cont...

GoldRet

	Percentiles	Smallest		
1%	-.0705071	-.0963523		
5%	-.0411961	-.0942303		
10%	-.0283615	-.0837025	Obs	833
25%	-.0115863	-.0821838	Sum of wgt.	833
50%	.0032998		Mean	.0019841
		Largest	Std. dev.	.0253551
75%	.0160616	.0911392		
90%	.0303873	.1063008	Variance	.0006429
95%	.0389286	.1191333	Skewness	-.0784091
99%	.0638143	.1319216	Kurtosis	5.432574

Figure 5: Sample Moment Summary of Weekly Gold Price Return 2005-2021

ApplReturn

	Percentiles	Smallest		
1%	-.1267848	-.5065874		
5%	-.0734749	-.2430599		
10%	-.0521449	-.195976	Obs	1,130
25%	-.0217912	-.1942862	Sum of wgt.	1,130
50%	.0076731		Mean	.0061534
		Largest	Std. dev.	.052576
75%	.0355935	.1719746		
90%	.0661697	.1983422	Variance	.0027642
95%	.0830427	.1986522	Skewness	-.8683249
99%	.1336532	.2664362	Kurtosis	12.41757

Figure 6: Sample Moment Summary of Weekly Apple Stock Return 2000-2021

Sample Autocorrelations

Definition: Sample Auto-Covariance Function

$$\hat{\gamma}_T(k) = \frac{1}{T-k} \sum_{t=k+1}^{T} (x_t - \bar{x}_T)(x_{t-k} - \bar{x}_T) \qquad \forall |k| < T$$

Sample Autocorrelations

Definition: Sample Auto-Covariance Function

$$\hat{\gamma}_T(k) = \frac{1}{T-k} \sum_{t=k+1}^{T} \left(x_t - \bar{x}_T\right)\left(x_{t-k} - \bar{x}_T\right) \qquad \forall |k| < T$$

Definition: Sample Auto-Correlation Function

$$\hat{\rho}_T(k) = \frac{\hat{\gamma}_T(k)}{\hat{\gamma}_T(0)} \qquad \forall |k| < T$$

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Definition: Sample Auto-Correlation Function

$$\hat{\rho}_T(k) = \frac{\hat{\gamma}_T(k)}{\hat{\gamma}_T(0)} \qquad \forall |k| < T$$

Under stationarity $\hat{\gamma}_T(k)$ consistently estimates $\gamma(k)$ as $T \to \infty$

Empirical Example Cont...

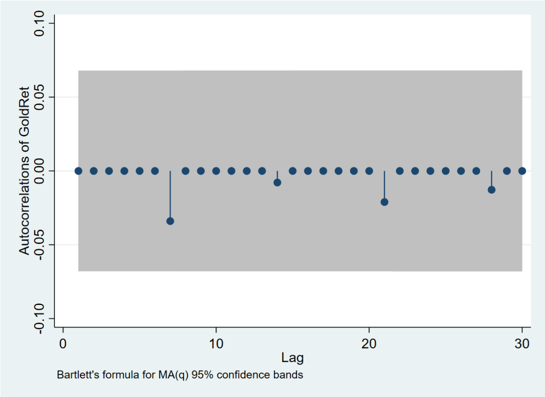

Figure 7: Sample Correlogram of Weekly Gold Price Return 2005-2021

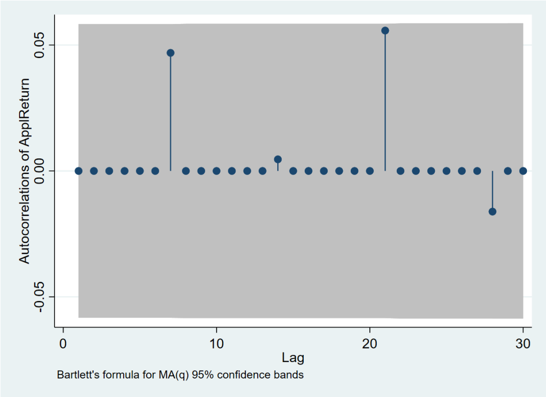

Figure 8: Sample Correlogramn of Weekly Apple Stock Return 2000-2021

Main Reading

B Chapter 1.1-1.5 [*A very gentle introduction to financial econometrics*]

B Appendix A1-A5 and **G** Appendix A1 and A2 and B1-B5 [Recap of some basic maths and stats results- **B** is a more basic introduction, those more advanced may skip to **G**]

H Chapters 1 and 2 [A more thorough intro to financial econometrics and properties of financial returns, leading onto next Week's material]

Some further discussion on the first four moments- mean, variance/standard deviation, skewness and kurtosis: http://rovdownloads.com/attachments/newsletters/Newsletter%2003%20-%20Understanding%20the%20Forecast%20Statistics%20and%20Four%20Moments%20(4P).pdf