Time Series Econometrics

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ECON5221: Problem Set 2

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Exercise Questions 2

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1. Consider a bivariate (i.e. two variable) VAR(1) system with $\phi_{11} = 0.6$, $\phi_{12} = 0.1$, $\phi_{21} = -0.6$, $\phi_{22} = 1.1$,

$$\mathbf{Y}_t = \boldsymbol{\alpha} + \Phi_1 \mathbf{Y}_{t-1} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\varepsilon}_t$ is vector white noise.

(a) Verify whether or not $\mathbf{Y}_t$ is a weakly stationary process

(b) If you are also given that

$$\boldsymbol{\alpha} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix},$$

find $\mathbb{E}[\mathbf{Y}_t]$.

(c) Obtain the values of the elements of the matrices Θ_s, for $s = 0, 1, 2$ (only) in the VMA representation

$$\mathbf{Y}_t = \boldsymbol{\mu} + \sum_{s=0}^{\infty} \Theta_s \boldsymbol{\varepsilon}_{t-s}.$$

(d) Using your results from (c), write down the first three values of the impulse response functions (corresponding to $s = 0, 1, 2$) for:

i. The effect of a unit shock to Y_{1t} on Y_{1t} and Y_{2t};

ii. The effect of a unit shock to Y_{2t} on Y_{1t} and y_{2t}.

,

2. Consider a stationary AR(2) process

$$Y_t = \mu_0 + \phi_{10} Y_{t-1} + \phi_{20} Y_{t-2} + \varepsilon_t \qquad \varepsilon_t \overset{i.i.d}{\sim} N(0, \sigma^2) \tag{1}$$

where a researcher observes a sample realisation of size $T > 2$, $(y_1, \cdots, y_T)$ from $\{Y_t\}$.

(a) Derive conditional log likelihood function for (1) conditioning on $(Y_1 = y_1, Y_2 = y_2)$.

(b) A researcher argues the conditional maximum likelihood estimator based on (a) will consistent even if ε_t is not normally distributed. Explain whether this statement is valid or not.

3. Consider an MA(1) process

$$Y_t = \mu_0 + \theta_{10} \varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \overset{i.i.d}{\sim} N(0, \sigma^2) \tag{2}$$

where a researcher observes a sample realisation of size T, $(y_1, \cdots, y_T)$ from $\{Y_t\}$.

(a) Derive conditional log likelihood function for (2) conditioning on $\varepsilon_0 = 0$.

(b) A researcher argues that we should not condition on $\varepsilon_0 = 0$ when $|\theta_{10}| > 1$. Discuss the problem caused in this scenario and how we could transform the model in (2) to remedy this.

4. Derive the autocovaiance function of the VMA(∞) process

$$\mathbf{Y}_t = \boldsymbol{\mu} + \sum_{s=0}^{\infty} \Theta_s \boldsymbol{\varepsilon}_{t-s} \qquad \varepsilon_t \sim WN(\Sigma).$$

where Σ is $k \times k$ var-covariance matrix of ε_t.