Econometric Time Series Analysis|EC5221

Wk5: Stationary Multivariate Time Series - (Orthoganlised) Impulse Response Functions

Nicky Grant (Semester 2, 2019/2020)

What We Will Cover Today

Recap: VAR(p) and their VMA(∞) form

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Impulse response functions

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Leftovers from previous lectures: **proof of consistency of sample mean with weakly-stationary data**

VMA(∞) Form of VAR(p)

$\Phi(L)\mathbf{Y}_t = \mathbf{c} + \boldsymbol{\varepsilon}_t$ where $\Phi(L) = (I_k - \Phi_1 L - \cdots - \Phi_p L^p)$

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\end{aligned}
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$A_0 = I_k$

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$A_0 = I_k$
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$$\begin{aligned} A_0 &= I_k \\ A_1 &= \Phi_1 \\ A_2 &= A_1\Phi_1 + \Phi_2 \end{aligned}$$

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$$\begin{aligned} A_0 &= I_k \\ A_1 &= \Phi_1 \\ A_2 &= A_1\Phi_1 + \Phi_2 \end{aligned}$$

i.e. $A_0 = I_k$ and $A_j = \sum_{s=0}^{j} A_s \Phi_{j-s}$ for $j = 1, 2, \cdots$

Impulse Response Functions

Impulse Response Functions: *Interpretation*

*VAR*s often have many coefficients & hence interactions difficult to interpret from these

Main interpretation tool is **impulse response function**

VAR(P) for $k \times 1$ $\mathbf{y}_t$:

$$\mathbf{Y}_t = \alpha + \Phi_1 \mathbf{Y}_{t-1} + ... + \Phi_P \mathbf{Y}_{t-P} + \varepsilon_t$$

Ask what is the effect of a "shock" to one variable on all in the system

Although relevant for VAR analysis, it is useful to fix idea in AR context.

Impulse Response Functions: AR(1)

Consider stationary AR(1) process:

$$Y_t = \phi Y_{t-1} + \varepsilon_t = \sum_{i=0}^{\infty} \theta_i \varepsilon_{t-i}, \qquad \text{for } \theta_i = \phi^i$$

What is impact on path of y if $\varepsilon_t \rightarrow (\varepsilon_t + \Delta\varepsilon_t)$?

Let $\{y_t, y_{t+1}, \ldots\}$ denote old path and $\{\tilde{y}_t, \tilde{y}_{t+1}, \ldots\}$ the new path.

in period t:

$$\tilde{y}_t = \phi y_{t-1} + \varepsilon_t + \Delta\varepsilon_t = y_t + \Delta\varepsilon_t$$

in period $t+1$:

$$\tilde{y}_{t+1} = \phi\tilde{y}_t + \varepsilon_{t+1} = \phi(y_t + \Delta\varepsilon_t) + \varepsilon_{t+1} = y_{t+1} + \phi\Delta\varepsilon_t$$

in period $t+2$:

$$\tilde{y}_{t+2} = \phi\tilde{y}_{t+1} + \varepsilon_{t+2} = \phi(y_{t+1} + \phi\Delta\varepsilon_t) + \varepsilon_{t+2} = y_{t+2} + \phi^2\Delta\varepsilon_t$$

Impulse Response Functions

In general (check for yourselves!):

$$\tilde{y}_{t+\ell} = y_{t+\ell} + \phi^{\ell}\Delta\varepsilon_t = y_{t+\ell} + \theta_\ell \Delta\varepsilon_t$$

$$\tilde{y}_{t+\ell} - y_{t+\ell} = \theta_\ell \Delta\varepsilon_t$$

If set $\Delta\varepsilon_t = 1$

$$\tilde{y}_{t+\ell} - y_{t+\ell} = \theta_\ell = \frac{\partial y_{t+\ell}}{\partial \varepsilon_t},$$ based on MA(∞) representation

$\frac{\partial y_{t+\ell}}{\partial \varepsilon_t}$ is known as impulse response at lag ℓ

$\frac{\partial y_{t+\ell}}{\partial \varepsilon_t}$ for $\ell = 1, 2, \ldots$ is known as the impulse response function

Impulse Response Functions: VAR(p)

Return to stationary $VAR(P)$ for $k \times 1$ $\mathbf{y}_t$:

$$\mathbf{Y}_t = \alpha + \Phi_1 \mathbf{Y}_{t-1} + \ldots + \Phi_P \mathbf{Y}_{t-P} + \varepsilon_t = \mu + \sum_{s=0}^{\infty} \Theta_s \varepsilon_{t-s}$$

Now track effects of shocks to $\varepsilon_{j,t}$ not only on $y_{j,t}$ but also on path of $y_{i,t}$.

The impulse response function for a unit "shock" to y_j on y_i is:

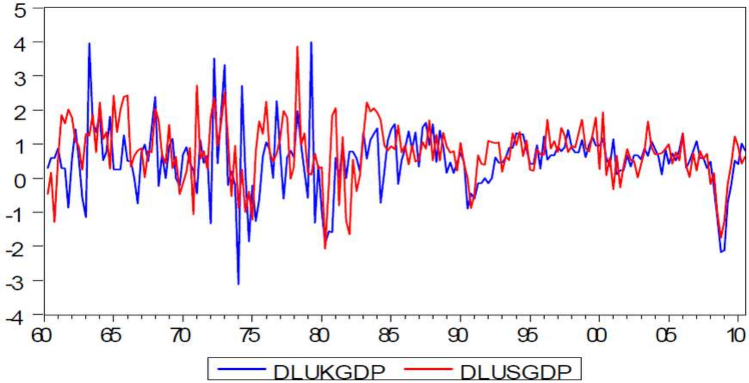
$$\frac{\partial y_{i,t}}{\partial \varepsilon_{j,t-s}} = \theta_{i,j}^{(s)}, \text{ for } s = 0, 1, 2, \ldots$$

where $\theta_{i,j}^{(s)}$ is $(i,j)^{th}$ element of Θ_s

Note:

$\theta_{i,j}^{(0)} = 1$ if $i = j$, $\theta_{i,j}^{(0)} = 0$ if $i \neq j$

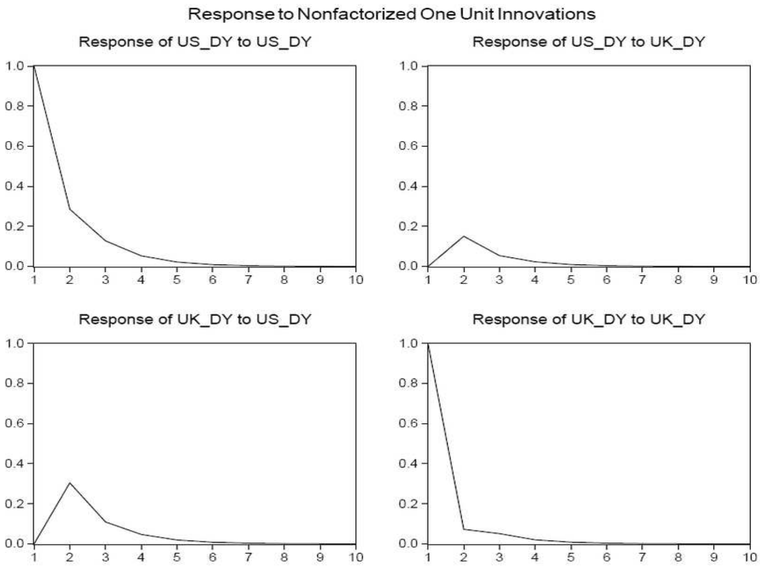
Empirical Example: VAR Estimation of UK/US GDP Growth



$$
\begin{aligned}
y_{us,t} &= \underset{(6.08)}{0.47} + \underset{(4.08)}{0.29} y_{us,t-1} + \underset{(2.34)}{0.15} y_{uk,t-1} + \widehat{u}_{us,t} \\
y_{uk,t} &= \underset{(3.11)}{0.27} + \underset{(3.88)}{0.30} y_{us,t-1} + \underset{(1.00)}{0.07} y_{uk,t-1} + \widehat{u}_{uk,t}
\end{aligned}
$$

$$
\widehat{\Sigma} = \begin{bmatrix} 0.65 & 0.20 \\ 0.20 & 0.81 \end{bmatrix} \Rightarrow \widehat{\rho}^{resid}_{us,uk} = 0.27
$$

Impulse Response Functions: UK/US GDP Growth



Orthogonalised Impulse Responses

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Conventional VAR: contemporaneous relationships in $E(\varepsilon_t \varepsilon_t') = \Sigma$

Impulse response functions above ignore covariances

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- Apply shock to individual equation (alone)
- Unrealistic if ε_t are **strongly correlated** across equations
- Orthogonalised impulse responses apply when "shocks" are uncorrelated

Orthogonalised Impulse Response Functions

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Orthogonalised impulse responses apply when "shocks" are uncorrelated

Orthogonalised impulse response function:
transform system so that new disturbances $\mathbf{u}_t$ have

$$E[\mathbf{u}_t \mathbf{u}_t'] = \Sigma_u \text{ diagonal}$$

Then "shock" for each equation is isolated

How to Orthogonalise?

VAR(p)

$$\mathbf{y}_t = \alpha + \Phi_1 \mathbf{y}_{t-1} + ... + \Phi_P \mathbf{y}_{t-P} + \varepsilon_t$$

Orthogonalised *VAR*, or structural *VAR* (*SVAR*):

Premultiply by matrix of constants **C** such that

$$\mathbf{C}\mathbf{y}_t = \mathbf{C}\alpha + \mathbf{C}\Phi_1 \mathbf{y}_{t-1} + ... + \mathbf{C}\Phi_P \mathbf{y}_{t-P} + \mathbf{C}\varepsilon_t$$

$$\mathbf{A}_0 \mathbf{y}_t = \alpha^* + \mathbf{A}_1 \mathbf{y}_{t-1} + ... + \mathbf{A}_P \mathbf{y}_{t-P} + \mathbf{u}_t$$

$$\mathbf{A}_0 = \mathbf{C},\ \alpha^* = \mathbf{C}\alpha,\ \mathbf{A}_j = \mathbf{C}\Phi_j\ (j = 1, ..., P),\ \mathbf{u}_t = \mathbf{C}\varepsilon_t$$

with $E[\mathbf{u}_t \mathbf{u}_t'] = \Sigma_u$ diagonal

There are many matrices **C** that achieve this

Most popular is the Cholesky decomposition

Cholesky Decomposition (Example)

For $k = 3$, $\mathbf{u}_t = \mathbf{C}\varepsilon_t \Rightarrow$

$$\begin{aligned}
\varepsilon_{1t} &= u_{1t} \\
c_{21}\varepsilon_{1t} + \varepsilon_{2t} &= u_{2t} \\
c_{31}\varepsilon_{1t} + c_{32}\varepsilon_{2t} + \varepsilon_{3t} &= u_{3t}
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Regress ε_{2t} on $\varepsilon_{1t} \Rightarrow E[\varepsilon_{2t}|\varepsilon_{1t}] = -c_{21}\varepsilon_{1t}$

Residual u_{2t} is component of ε_{2t} not correlated with ε_{1t}
$\Rightarrow$ u_{2t} is uncorrelated with $u_{1t} = \varepsilon_{1t}$

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Regress ε_{3t} on ε_{1t} & ε_{2t}: $E[\varepsilon_{3t}|\varepsilon_{1t}, \varepsilon_{2t}] = -c_{31}\varepsilon_{1t} - c_{32}\varepsilon_{2t}$

Residual u_{3t} is component of ε_{3t} not correlated with ε_{1t} or ε_{2t}
$\Rightarrow$ u_{3t} is uncorrelated with $u_{1t} = \varepsilon_{1t}$ & $u_{2t} = c_{21}\varepsilon_{1t} + \varepsilon_{2t}$

Sequential procedure generalises to any k

Cholesky Decomposition: Interpretation/Assumptions

Matrix **C** is applied to entire VAR
not just disturbances

$VAR(1)$ with $k = 3$: Orthogonalised system has form

$$\begin{bmatrix} 1 & 0 & 0 \\ c_{21} & 1 & 0 \\ c_{31} & c_{32} & 1 \end{bmatrix} \mathbf{y}_t = \begin{bmatrix} 1 & 0 & 0 \\ c_{21} & 1 & 0 \\ c_{31} & c_{32} & 1 \end{bmatrix} \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \mathbf{y}_{t-}$$
$$+\alpha^* + \mathbf{u}_t$$

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$$+\alpha^* + \mathbf{u}_t$$

$\Rightarrow$ contemporaneous causal ordering y_{1t}, y_{2t}, y_{3t}

Variables ordered lower assumed not to affect those above at t
Has structural economic interpretation;
often referred to as structural VAR ($SVAR$)

Lag coefficients also changed by the transformation

Cholesky Decomposition: Interpretation/Assumptions

Different ordering of variables yields different *SVAR*

- Hence different orthogonalised impulse response functions
- Ordering not innocuous!

Cholesky Decomposition: Interpretation/Assumptions

Different ordering of variables yields different *SVAR*

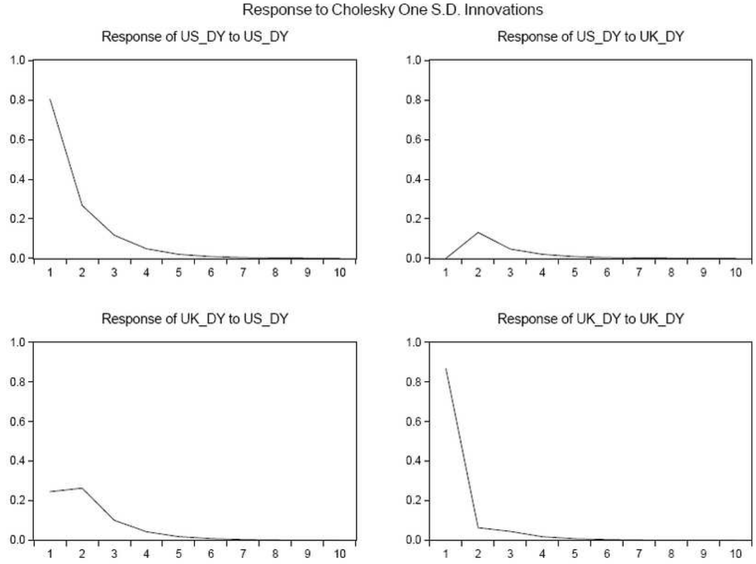
- Hence different orthogonalised impulse response functions
- Ordering not innocuous!

Cholesky orthogonalised impulse responses assume specific contemporaneous causal ordering

- Ideally given by economic theory

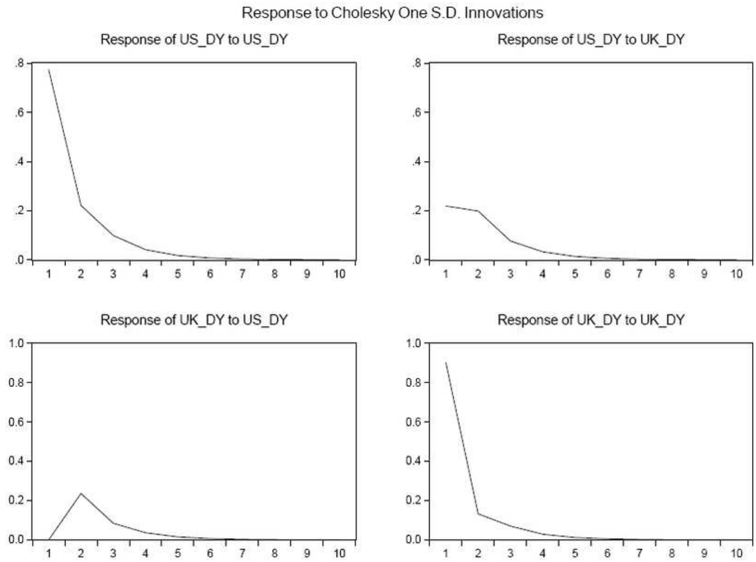
Cholesky Decomposition:Empirical Example

Order US first in US/UK growth VAR



Cholesky Decomposition:Empirical Example

Now order UK first in US/UK VAR



Impulse Response Case Study

Empirical Example: How have world shocks affected the UK economy?

Chowla, Quaglietti and Rachel, "**How have world shocks affected the UK economy**?", *Bank of England Quarterly Bulletin* 2014 Q2 issue.

Their article analyses two issues:

- how have world shocks affected the UK economy since the onset of Financial Crisis in 2007?
- through what economic channels have these effects taken place?

Empirical Example: How have world shocks affected the UK economy?

CQR use VAR to model $\mathbf{y}_t$ consisting of two blocks of variables.

$$\mathbf{Y}^{W} = \begin{bmatrix} \text{growth rate of world GDP} \\ \text{growth rate of world prices} \\ \text{US interest rate spread} \\ \text{VIX index} \end{bmatrix}$$

$$\mathbf{Y}^{UK} = \begin{bmatrix} \text{growth rate of UK GDP} \\ \text{growth rate of CPI} \\ \text{Bank rate} \end{bmatrix}$$

Empirical Example: How have world shocks affected the UK economy?

Ordering: World then UK so that

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{W} \\ \mathbf{Y}^{UK} \end{bmatrix}$$

Using quarterly data 1987.Q1-2013.Q4, estimate VAR(2) model

$$\mathbf{Y}_t = \mu + \Phi_1 \mathbf{Y}_{t-1} + \Phi_2 \mathbf{Y}_{t-2} + \varepsilon_t$$

to obtain fitted model:

$$\mathbf{y}_t = \hat{\mu} + \hat{\Phi}_1 \mathbf{y}_{t-1} + \hat{\Phi}_2 \mathbf{y}_{t-2} + \hat{\varepsilon}_t$$

Empirical Example: How have world shocks affected the UK economy?

To construct Chart 3, CQR need to estimate what part of the shocks $\{\varepsilon_t\}$ can be attributed to world events

Assume $\{\varepsilon_t\}$ generated from a vector of "structural" (**orthogonalized**) shocks $\mathbf{u}_t$ via:

$$\mathbf{u}_t = C\varepsilon_t$$

where C is a lower triangular matrix associated with **Cholesky decomposition.**

Empirical Example: How have world shocks affected the UK economy?

Due to variable ordering:

$u_{5,t}$, the structural shock to UK GDP growth is the part of $\varepsilon_{5,t}$ that cannot be explained by the shocks to the world variables.

$u_{6,t}$, the structural shock to UK CPI is the part of $\varepsilon_{6,t}$ that cannot be explained by the world shocks or the shock to UK GDP growth.

$u_{7,t}$, the structural shock to the UK Bank rate is the part of $\varepsilon_{7,t}$ that cannot be explained by the world shocks or the shocks to UK GDP growth or UK CPI growth.

Empirical Example: How have world shocks affected the UK economy?

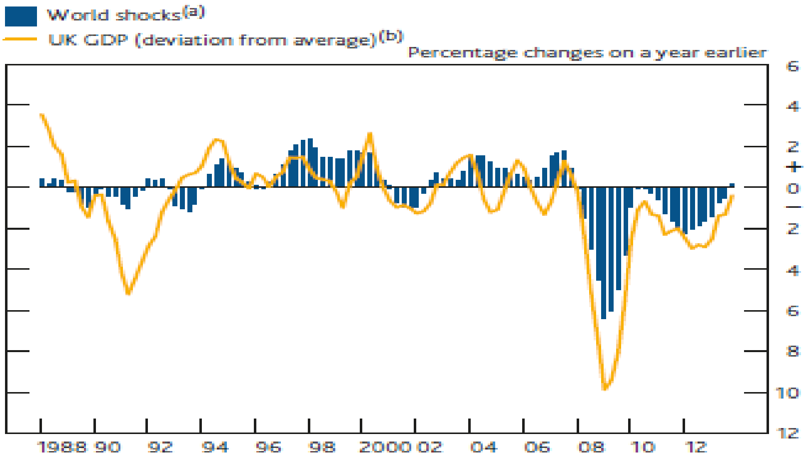
- *Step 1:* $\hat{\mathbf{u}}_t = C\hat{\varepsilon}_t$ for $t = 1, 2, \ldots T$.
- *Step 2:* $\tilde{\mathbf{u}}'_t \;=\; [\,\hat{u}_{1,t}, \hat{u}_{2,t}, \hat{u}_{3,t}, \hat{u}_{4,t}, 0, 0, 0\,]$.
- *Step 3:* $\tilde{\varepsilon}_t = C^{-1}\tilde{\mathbf{u}}_t$.
- *Step 4:* Construct $\tilde{\mathbf{y}}_t$ via:

$$\tilde{\mathbf{y}}_t \;=\; \hat{\mu} + \hat{\Phi}_1\,\tilde{\mathbf{y}}_{t-1} + \hat{\Phi}_2\,\tilde{\mathbf{y}}_{t-2} + \tilde{\varepsilon}_t, \qquad t \;=\; 3, 4, \ldots, T$$

for $\tilde{\mathbf{y}}_1 = \mathbf{y}_1$, $\tilde{\mathbf{y}}_2 = \mathbf{y}_2$.

Empirical Example: How have world shocks affected the UK economy?

Chart 3 Estimates of the historical impact of world shocks on UK activity



Sources: Bloomberg, Bureau for Economic Policy Analysis, IMF, OECD, ONS, Thomson Reuters Datastream and Bank calculations.

Main Reading

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Hamilton Chapter 11.4, and 11.6 (pp. 326-33)

Luktepohl Section 2.3.2.