ECON61001: Econometric Methods

Exercise 5: Robust Inference- Heteroscedasticity and Serial Correlation

Nicky Grant (2014)

Class - Diagnostic Testing and Robust Inference

Question 1 regards testing for heteroskedasticity using the White Test, and also a test specifically for heteroskedasticity of a linear form and performing robust inference on parameter restriction tests

1. A colleague has estimated the following regression model:

$$y_i = \alpha_0 + \alpha_1 x_i + u_i \tag{1}$$

for $i = 1, .., 1000$ and obtained OLS parameter estimates for α_j, $(j = 0, 1.)$ Let $\hat{u}_i$ be the estimated residual based on the OLS estimates of α_j $(j = 0, 1)$.

Suppose you have results from,

$$\widehat{u}_i^2 = \eta_0 + \eta_1 x_i + error \tag{2}$$

where $R^2 = 0.011$ and,

$$\widehat{u}_i^2 = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + error \tag{3}$$

where $R^2 = 0.12$.

a) Outline how (2) and (3) may be used to test the assumption that u_i is homoscedastic. Provide details on the test statistic and also the distribution used to perform such a test.

b) Using your preferred regression (2) and (3) perform a test at the 5% significance level that u_i is homoscedastic. Comment on your choice of preferred test and discuss the result of the test.

c) A researcher now runs the regression in (1) and finds,

$$\underset{\substack{(1.216)\\ [1.362]}}{\widehat{y}_i = 0.389} + \underset{\substack{(0.927)\\ [1.602]}}{1.896 x_i} \tag{4}$$

where (normal standard errors in parentheses ( ) and White Standard Errors standard errors in square brackets [ ]).

i. Using (4) test the following hypothesis (at a 95% significance level):

$$H_0 : \alpha_0 = 0; H_A : \alpha_0 > 0$$

stating which standard errors are used to perform the test along with your reasoning. Ensure that you clearly state the respective test statistics, their distributions, the decision rules and calculations.

ii. Another researcher suggests inference in (4) would be more accurate using Newey West Standard Errors. Should the researcher take this advice, given she finds no evidence of serial correlation in u_i?

Question 2 is about the impact of heteroscedasticity on unbiasedness and efficiency of OLS and constructing an efficient estimator using Weighted Least Squares.

2. Consider the linear model,

$$y_i = x_i' \beta_0 + u_i \tag{5}$$

where $x_i = (x_{1i}, x_{2i}, .., x_{ki})'$ is a $k \times 1$ vector and define $y = (y_1, .., y_n)$, $X = (x_1, ..x_n)'$, $u = (u_1, .., u_n)'$.

$$u_i = \eta_i x_{1i} \tag{6}$$

Where $E[\eta_i|X] = 0$, $E[\eta_i^2|X] = \sigma_\eta^2$, $E[\eta_i \eta_j|X] = 0$ for $i \neq j$.

a) Show that OLS is unbiased.

b) Derive $Var(\hat{\beta}|X)$, is OLS efficient?

c) Suggest an estimator that is unbiased and efficient and show the variance of this estimator.

d) Suppose $u_i = \eta_i x_{2i}$, is the estimator in (c) efficient?

Question 3 regards testing for serial correlation and robust inference with serial correlation.

3. You have estimated the following regression relationship

$$y_t = \alpha + \beta x_t + \gamma z_t + \delta w_t + u_t$$

and obtained the following set of results

$$\widehat{y}_t = \underset{\substack{(0.524)\\ [0.617]\\ \{0.652\}}}{0.835} + \underset{\substack{(1.215)\\ [1.387]\\ \{1.482\}}}{0.721}\, x_t - \underset{\substack{(0.315)\\ [0.472]\\ \{0.523\}}}{1.581}\ z_t + \underset{\substack{(0.003)\\ [0.004]\\ \{0.005\}}}{0.023}\ w_t$$

$$T = 412,\ R^2 = 0.624,$$

$$RSS = 384.88,\ F - Stat = 582.46, \widehat{\sigma}_\varepsilon = 0.943$$

where the values below the parameter estimates are: (normal standard errors), [White se] and {Newey-West se}.

a) Under what circumstances would you use Newey-West standard errors to calculate t-tests? Does the sample size play any role in your argument?

b) You also have the following result from an auxilliary regression

$$\widehat{u}_t = \eta_1 + \eta_2\ x_t + \eta_3\ z_t + \eta_4\ w_t + \eta_5\ \widehat{u}_{t-1} + error$$

$$T = 411,\ R^2 = 0.0182.$$

Which assumption can be tested with this information? Test this assumption and subsequently test the following hypotheses (use $\alpha = 0.05$):

$$H_0 \ : \ \beta = 0.5; H_A : \beta \neq 0.5$$

$$H_0 \ : \ \gamma \geq 0; H_A : \gamma < 0.$$

Video - Diagnostic Testing and Robust Inference

These questions will not be covered in class, solutions will be provided and also an online clip by Ralf Becker goes over these solutions. `https://www.youtube.com/watch?v=msv40k_IsMQ`. Note the solutions give details on MATLAB. For those not interested you can just listen to the solution to the question with the MATLAB background. Written solutions will also be given. Please attempt these questions before looking at the solution. Note question 2 in the online video solution is no longer covered in this course.

Video Question 1 is about testing for HS/AC using various tests.

1. Below you can find a range of auxilliary regressions testing for either autocorrelation (AC) or heteroskedasticity (HS) on some regression residuals u_t. For each test complete the missing information (p-values only approximations if using Tables; d.o.f. = degrees of freedom, use $\alpha = 0.01$):

$$\begin{aligned} A &: \widehat{u}_t^2 = \alpha_0 + \alpha_1 x_t + \alpha_2 x_t^2 + \alpha_3 z_t + \alpha_4 z_t^2 + v_t \\ AC \text{ or } HS? \qquad n &= 45;\ R^2 = 0.089;\ Test - Stat = \qquad ; \\ d.o.f. &= \qquad ; p - value = \qquad ;\ \text{Reject } H_0? \end{aligned}$$

$$\begin{aligned} B &: \widehat{u}_t = \alpha_0 + \alpha_1 x_t + \alpha_2 \widehat{u}_{t-1} + \alpha_3 \widehat{u}_{t-2} + v_t \\ AC \text{ or } HS? \qquad n &= 247;\ R^2 = 0.040;\ Test - Stat = \qquad ; \\ d.o.f. &= \qquad ; p - value = \qquad ;\ \text{Reject } H_0? \end{aligned}$$

$$\begin{aligned} C &: \widehat{u}_t^2 = \alpha_0 + \alpha_1 z_t + \alpha_2 x_t + v_t \\ AC \text{ or } HS? \qquad n &= 135;\ R^2 = \qquad ;\ Test - Stat = \qquad ; \\ d.o.f. &= \qquad ; p - value = 0.05;\ \text{Reject } H_0? \end{aligned}$$

$$\begin{aligned} D &: \widehat{u}_t = \alpha_0 + \alpha_1 z_t + \alpha_2 x_t + \alpha_3 \widehat{u}_{t-1} + v_t \\ AC \text{ or } HS? \qquad n &= \qquad ;\ R^2 = 0.006498;\ Test - Stat = \qquad ; \\ d.o.f. &= \qquad ; p - value = 0.01;\ \text{Reject } H_0? \end{aligned}$$

2. Do not attempt

   Video Question 2 demonstrates why we cannot use a similar argument to the white test, replacing $E[u_i u_j|\mathbf{X}]$ with $\hat{u}_i\hat{u}_j$ as mentioned in lecture.

3. Assume, in order to calculate NW s.e., Ω was estimated with $\widehat{\mathbf{u}}\widehat{\mathbf{u}}'$, where $\widehat{\mathbf{u}}$ is a vector of the estimated OLS residuals. Explain why this would result in $Var\left(\widehat{\beta}_{OLS}|\mathbf{X}\right) = 0$.