# Intro to Hypothesis Testing on the Population Mean

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# Motivation

**Student A** claims the average economics grade in the UK population of UGs is no more than 65%

**Student B** disagrees believing this average grade is larger than $65\%$

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**Null Hypothesis:** $H_0 : \mu <= 65$

**Alternative Hypothesis:** $H_A : \mu > 65$

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**Q: If the mean in our sample was 67% does this imply Student A is incorrect?**

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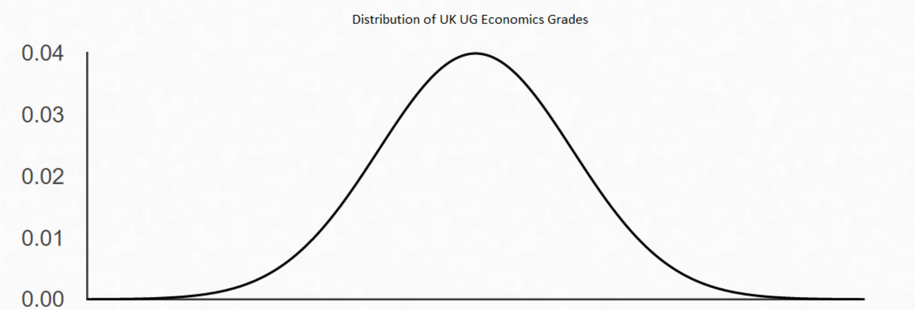


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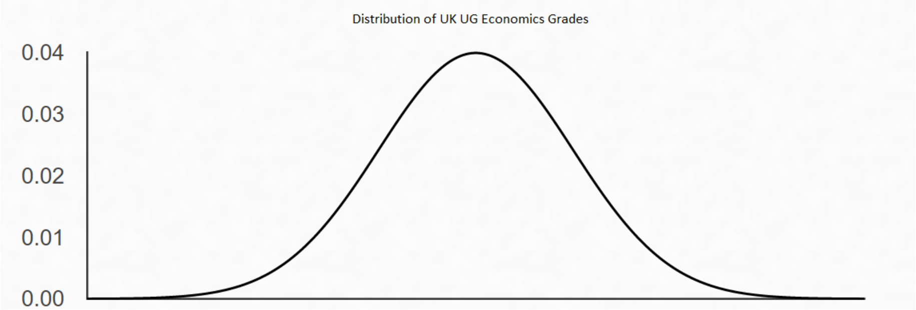


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**Sample Mean Distribution**: $\hat{\mu}$ is normally distributed with mean $\mu$ and variance $\frac{\sigma^2}{25} = 4$

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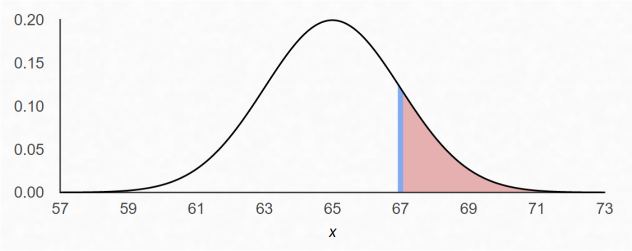


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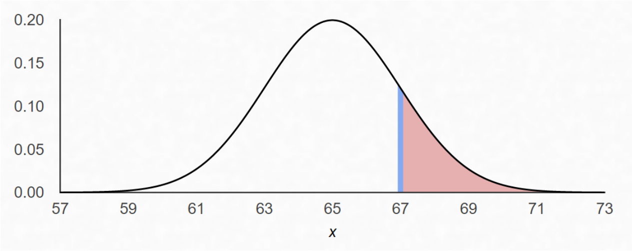


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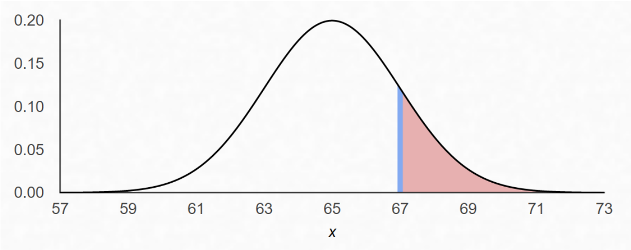


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Both probabilities are known as **p-values** you may have seen reported on statistical software

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**Test Procedure: Accept** $H_0$ if $\hat{\mu} <= 68.29$ and **Reject** if $\hat{\mu} > 68.29$

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$\frac{\hat{\mu}-\mu}{\sigma/\sqrt{N}}$ is the **standardised sample mean** (**z-stat**)

z-stat is distributed standard normal (mean 0 variance 1)

Perform test of Null Hypothesis by comparing z-stat with critical value from standard normal distribution tables

## Standardised Test

$\mu + 10$ $\mu + 20$ $\mu + 30$ $\mu - 10$ $\mu - 20$ $\mu - 30$